

# Optimal Debt and Equity Values in the Presence of Chapter 7 and Chapter 11

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## ABSTRACT

Explicit presence of a reorganization process in addition to liquidation can lead to conflicts of interest between borrowers and lenders. In the first-best outcome, the reorganization adds value to both parties via higher debt capacity, lower credit spreads, and improvement in the overall firm value. If control of the ex-ante timing of reorganization and the ex-post decision to liquidate is given to borrowers, most of the benefits of the code are appropriated by borrowers ex-post. Lenders can restore the first-best outcome by seizing this control or by the ex-post transfer of control rights. On average, firms are more likely to default and are less likely to liquidate relative to the benchmark case with liquidation only.

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The U.S. bankruptcy code, which includes a liquidation process (Chapter 7), and a reorganization process (Chapter 11), aims to resolve a number of important issues associated with a distressed firm. These issues can be classified either into information asymmetry (e.g., quality of the firm, Heinkel and Zechner, 1993), agency problems (e.g., risk shifting, Jensen and Meckling, 1976), or coordination problems (e.g., debt of various maturities, Berglof and von Thadden, 1994). In this paper, we investigate if there is a place for a reorganization process in the presence of costly financial distress and liquidation. These costs capture, in reduced form, the aforementioned frictions. We wish to characterize the states of a borrowing firm relative to its outstanding contractual debt obligations at different stages of financial distress assuming full information and a single issue of debt. We also wish to determine the role played by the bankruptcy code in improving the welfare of borrowers and lenders and how its influence depends on the rights given to borrowers and lenders at various stages of financial distress.

More precisely, we build on the framework of structural modeling of debt (Merton, 1974; Black and Cox, 1976), which allows us to determine the overall firm value, along with the values of equity and debt. In line with this literature, we assume that liquidation destroys part of the firm's value. We ask the basic and yet important question whether there is any value in the availability of Chapter 11 even in the absence of asymmetric information, agency costs, and coordination problems.<sup>1</sup> Put differently, we try to determine if a distressed firm would choose the reorganization option under nearly perfect market assumptions. Our model extends the work of Leland (1994) who exclusively characterizes the liquidation option (Chapter 7). We superimpose a sparse characterization of Chapter 11 aimed to preserve the critical characteristics of the process, such as automatic stay, grace period, absolute priority, and transfer of control rights from equity to debt in bad states.<sup>2</sup>

In deciding which features of the reorganization proceedings must be incorporated in our stylized model, we follow Hart (1999) who identifies the goals of an efficient bankruptcy procedure which enjoy a broad consensus in the theoretical and empirical work on the subject. First, a good bankruptcy procedure should deliver an ex-post efficient outcome. Intuitively, this translates into total firm value maximization, ex-post. Our model incorporates this feature by solving for the first best outcome and, subsequently, addresses the question of achieving this outcome ex-post by shift-

ing control from equity to debt. Second, the bankruptcy code should provide incentives to make contractual payments in more states of the world by penalizing equity holders and managers in bankruptcy states.<sup>3</sup> Absent this feature there will be typically no lending, ex-ante. Suspension of dividends and the imposition of absolute priority rules (APR) upon entering bankruptcy are very much in the spirit of this goal. We explicitly track the suspended dividends in our model and use them to repay the coupons accumulated in arrears as a result of the automatic stay provision. We also enforce APR by giving all residual value to creditors upon liquidation. Third, to provide appropriate incentives when in Chapter 11, some part of value should be set aside for shareholders if the firm can avoid liquidation and re-emerge from bankruptcy. This goal is modeled by allowing debt forgiveness once in Chapter 11.

Our modeling approach allows us to draw a clear distinction between the notions of bankruptcy and liquidation. Absent the reorganization option, Black and Cox (1976) and Leland (1994) have shown that the firm should issue equity to finance its contractual debt obligations until the equity value is driven to zero. This rule implies that the firm is not bankrupt as long as its equity value is positive. This rule also endogenously determines the bankruptcy boundary which coincides with the liquidation boundary. There is no room for bankruptcy without liquidation because the firm meets its contractual obligations either from operating cash flows, or by issuing equity until it is liquidated. There is no state in which the equity values are positive, but the firm, nevertheless, rationally chooses not to fulfill its obligations.<sup>4</sup> This intermediate state prior to liquidation is introduced in our model by the presence of reorganization under Chapter 11. Thus, we can define a firm as being bankrupt when it files for Chapter 11, i.e., its operating cash flows are below an additional (endogenously determined) boundary even though its equity value is positive.

The conclusions regarding welfare improvement that emerge from our model are nuanced: at a first glance, one might expect that the addition of another outside option in the form of a reorganization besides the liquidation option should lead to an improvement of welfare for everyone. That is certainly not the case, in general. Equity value maximization is no longer consistent with total firm value maximization in the presence of limited liability (in contrast to Leland, 1994), when the agents are presented with the options to reorganize or to liquidate. This result reflects the fact that when debtors are given the right to decide when to file for Chapter 11, they attempt to capture the

rents associated with the additional option, such as debt forgiveness and suspension of contractual payments, by filing too early.<sup>5</sup> Early default leads to the decline in the overall firm value relative to the first best scenario in both, the Leland and our, models. Debt forgiveness may also be in the interest of the lenders and the firm as a whole because costly liquidation could be avoided. Nonetheless total firm value maximization generally requires filing for Chapter 11 later (relative to the equity value maximization), because this extends the period of complete contractual payments by the debtors. The divergence between firm value maximization and equity value maximization brings up some important issues that are absent in the benchmark model of Leland (1994) and, more generally, in the corporate debt literature (to the best of our knowledge).

We address the issue of whether the “first best” (i.e., the one that maximizes the total firm value subject to the limited liability of equity and debt) outcome can be restored. We show that debt maximization leads to a strategy that is qualitatively very similar to the first best. Therefore, one way in which the first best can be restored is by “contingent transfer of control rights” once the Chapter 11 decision is made by the debtors. This feature has been stressed in the incomplete contracting literature (see Aghion and Bolton, 1992 and Dewatripont and Tirole, 1994, for example). We allow creditors to take control of the reorganization process by choosing the length of the grace period. This feature will induce debtors to file for Chapter 11 later because the length of the grace period introduces a tradeoff between a credible liquidation threat (preferred by creditors) and the ability to recover from distress (preferred by debtors).

Irrespective of who has control over the decision making process, the probabilities of default increase relative to the Leland benchmark. This is natural since bankruptcy in our model occurs no later than the bankruptcy/liquidation in the Leland model. Therefore, separating different forms of distress allows us to ameliorate the shortcomings of the extant models, which tend to underestimate the default probabilities at short horizons. One might argue that this result is built in mechanically by introducing a new endogenous boundary. However, the probability of filing for Chapter 11 is what practitioners are concerned with. This probability is closely linked to the popular expected default frequency (EDF) used by Moody’s|KMV (Crosbie and Bohn, 2002; Leland, 2004). Interestingly, once the firm value is so low that it files for Chapter 11 in our model, the probability of liquidation declines in many states of the world relative to the Leland model. Hence, though

our model induces earlier bankruptcy, the bankruptcy mechanism can decrease the probability of liquidation.

The paper is organized into six sections. The next section presents the details of our model, discusses valuation issues and contrasts the model with the related work. Section II discusses the model implications under the first best scenario. Section III contrasts debt and equity maximization and establishes that debt maximization is very close to the first best. Section IV is concerned with transfer of control rights from equity to debt in order to achieve the first best. Section V discusses the impact of the optimal debt level on the results. Conclusions are given in the final section and computational methodology is presented in the appendix.

## I. Model of Default and Liquidation

### A. The Setup

We fix the filtered probability space  $(\Omega, \{\mathcal{F}_t\}, \mathcal{F}, \mathbb{P})$  and, following Goldstein, Ju, and Leland (2001), we choose our primitive variable,  $\delta_t$ , to be the operating cash flows or earnings before interest and taxes (EBIT). The EBIT process under the risk-neutral measure  $\mathbb{Q}$  is given by:

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dW_t(\mathbb{Q}). \quad (1)$$

This assumption implies that the unlevered value of the firm's asset (or, a claim on the entire payout) is equal to:

$$V_t = \mathbf{E}^{\mathbb{Q}} \left( \int_t^{\infty} e^{-r(s-t)} \delta_s ds \mid \mathcal{F}_t \right) = \frac{\delta_t}{r - \mu}. \quad (2)$$

Note that  $\mu < r$ . Since  $\mu$  is a constant (2) implies:

$$\frac{dV_t}{V_t} = \mu dt + \sigma dW_t(\mathbb{Q}) \quad (3)$$

Note that the risk-adjusted growth rate of earnings  $\mu$  coincides with the risk-adjusted growth rate of the unlevered value.

The firm raises cash to finance its projects by issuing one consol bond. As a result, the coupon rate  $c$  determines the firm's capital structure. We assume that potential creditors, and the firm's

management/equity holders, have full information about the EBIT characteristics  $\mu$  and  $\sigma$ . Also, they are able to observe the values of EBIT continuously. There is no meaningful role for debt in this setting unless there are corporate taxes and costly bankruptcy (Modigliani and Miller, 1958; Modigliani and Miller, 1963; Kraus and Litzenberger, 1973). We assume that the coupon rate was selected optimally taking into account these frictions. However, since the tax advantages of debt are obvious for the firm's equity holders, we ignore taxes to emphasize the impact of bankruptcy. In Section V we return to this issue and explicitly consider the optimal capital structure in the presence of taxes and bankruptcy costs.

We assume that the investment policy of the firm is fixed, and EBIT is used to pay off the debt. If  $\delta_t - c > 0$  we say the firm is in the liquid state and  $\delta_t - c$  is distributed to the equity holders of the company as dividends. When earnings become less than the promised (contractual) coupon obligations, the firm has to find a way to generate extra cash.

In the Leland (1994) model the firm issues additional equity to meet the coupon payments. It can do so until the equity value becomes equal to zero. In our model, the firm may choose to default (Chapter 11) prior to completely destroying the equity value. This decision may still lead to liquidation (Chapter 7), or it may result in recovery from default. We model these possibilities by introducing two (potentially endogenous) barriers for  $V_t$ . The first barrier  $V^B$  (with the corresponding value of EBIT  $\delta^B = V^B(r - \mu)$ ) determines the Chapter 11 filing, and the second barrier  $V^L$  (with the corresponding value of EBIT  $\delta^L = V^L(r - \mu)$ ) leads to liquidation. This feature is the point of qualitative departure from Leland (1994), who allows for only one, liquidation, boundary.

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It is important to point out at this stage that we are modeling financial rather than economical distress. Therefore, bankruptcy by itself is not going to cause poor performance.<sup>7</sup> The firms that we consider are simply illiquid. We model this feature by keeping the same process for EBIT before and after bankruptcy. Figures 1 and 2 illustrate further events.

[Figures 1 and 2 are about here.]

In practice, prior to default, the firm has an option to restructure its debt either through a private workout in case of the bank debt, or an exchange offer in case of public debt.<sup>8</sup> We focus on the value of the bankruptcy option. Therefore, while our paper will have nothing to say about the optimal choice between the different workout alternatives, our results will nevertheless provide the benchmark for making this decision.

Typical bond indentures stipulate that creditors have an absolute and unconditional right to receive payment of principal and accrued interest (if any and specified) on the bonds once the company defaults.<sup>9</sup> In order to protect the firm from creditors, the automatic stay provision takes effect for the duration the exclusivity period of 120 days, which is often extended. In particular, this means that interest payments stop on all unsecured debt, effectively extending the maturity of all firm's debt obligations. We model the automatic stay provision recording the accumulated unpaid coupons plus interest in arrears in the variable  $A_t$ .<sup>10</sup>

The firm may return to the liquid state at some future time  $T$ , i.e.,  $V_T = V^B$ , (this scenario corresponds to path  $A$  in Figure 1) and pay the debtholder  $\theta A_T$ , where  $0 \leq \theta \leq 1$ .<sup>11</sup> Effectively, the creditors will forgive a fraction  $1 - \theta$  of the arrears. This modeling strategy reflects the desired goal that equity holders also get something in the Chapter 11 process. The parameter  $\theta$  could be viewed as a reduced-form outcome of a bargaining game between debt and equity similar to Fan and Sundaresan (2000).<sup>12</sup> For simplicity, we assume that  $\theta$  is a constant.

In part, the feasibility of resurfacing from default will be determined by the ability of the firm to repay the accumulated arrears. In our model, the firm does not pay dividends to the shareholders while it is in default. Instead the entire EBIT while in default is accumulated in a separate account and the amount is denoted  $S_t$ . If the firm emerges from default at time  $T$ , it must pay the amount of the arrears owed,  $\theta A_T$ , to the creditors. This requirement can be satisfied in one of two ways. If  $S_T \geq \theta A_T$  then the entire amount in this account is applied towards the arrears. Any leftover is distributed to the shareholders. If  $S_T$  does not contain an amount sufficient to repay the arrears, the equity is diluted to raise the remaining amount.<sup>13</sup>

While in bankruptcy, the firm will be exposed to the continuously accruing proportional distress cost  $\omega$ . This cost can represent legal fees, lost business and the loss of valuable employees. This

cost insures that the firm would not want to drag out the bankruptcy forever. It also implies that bankruptcy results in some economic distress as well.

Finally, the company is allowed to stay in Chapter 11 for no more than the duration of a grace period  $d$ . If the company spends more time than  $d$  in default (path  $B$  in the Figure 1), or if the value of unlevered assets reaches  $V^L$  (path  $C$ ) the firm liquidates with proportional cost  $\alpha$ .<sup>14</sup> The first condition matches the real-life practice of the bankruptcy judge removing the automatic stay provision after the current exclusivity period has ended and creditors are not willing to extend it. The second condition is equivalent to the equity value falling to zero as in the Leland (1994) model.

### *B. Determining Equity, Debt, and Firm Values*

Given our setup, we can value the equity and the debt of the firm. We follow Bielecki and Rutkowski (2002) in computing these values via the risk-neutral valuation, or martingale, approach. In order to succinctly describe the details of our valuation, we need to describe the evolution of the objects of interest mathematically and to introduce additional notation. Table I contains a summary of all notation.

[ Table I is about here.]

The arrears  $A_t$  evolve according to:

$$dA_t = \begin{cases} rA_t dt + c dt & \text{if } V^L \leq V_t < V^B \\ -A_{t-} & \text{if } V_t = V^B \\ 0 & \text{if } V_t > V^B \end{cases} . \quad (4)$$

Thus the arrears account resets to zero upon clearing bankruptcy at time  $t$  when  $V_t$  reaches  $V^B$  from below. In addition, we model the accumulation of earnings in the default region via the process  $S_t$ :

$$dS_t = \begin{cases} rS_t dt + \delta_t dt & \text{if } V^L \leq V_t < V^B \\ -S_{t-} & \text{if } V_t = V^B \\ 0 & \text{if } V_t > V^B \end{cases} . \quad (5)$$

The resetting of  $A_t$  and  $S_t$  to zero at  $V^B$  implies that these processes are discontinuous. In order to avoid notational ambiguities we take their sample paths to be right continuous with left limits, i.e., càdlàg.

Let

$$\tau_t^{V^B} = \sup\{s \leq t : V_s = V^B\} \quad (6)$$

be the most recent time the firm reached the bankruptcy boundary (we assume that  $\tau_t^{V^B} = \infty$  if there was no bankruptcy before time  $t$ ). One of the reasons the firm can liquidate is that it spent too much time (longer than  $d$ ) under the bankruptcy boundary. In this case, the time of liquidation can be computed as:

$$\tau_t^d = \inf\{s \geq t : s - \tau_s^{V^B} \geq d, V_s \leq V^B\}. \quad (7)$$

The other reason the firm liquidates is that the firm value simply reaches the liquidation value  $V^L$ . Therefore, liquidation can occur at time  $\tau_t^{V^L}$ , where

$$\tau_t^{V^L} = \inf\{s \geq t : V_s = V^L\}. \quad (8)$$

Therefore, the liquidation time is the smallest of the two:

$$\mathcal{T}_t = \tau_t^d \wedge \tau_t^{V^L} \wedge \infty. \quad (9)$$

Now, given the definition of the liquidation time, we can determine the total equity value as:

$$E(t, \delta_t, \tau_t^{V^B}, \tau_t^{V^L}) = \mathbf{E}^{\mathbb{Q}} \left\{ \int_t^{\mathcal{T}_t} e^{-r(s-t)} \left[ (\delta_s - c) \mathbf{1}_{\{V_s \geq V^B\}} - \omega V_s \mathbf{1}_{\{V^L < V_s < V^B\}} + (S_{s-} - \theta A_{s-}) \delta^{\mathcal{D}}(s - \tau_t^{V^B}) \right] ds | \mathcal{F}_t \right\}, \quad (10)$$

where  $\mathbf{1}_{\{x\}}$  is the indicator function of the event  $x$ , i.e., is one if  $x$  is true and zero otherwise, and  $\delta^{\mathcal{D}}(x)$  is the Dirac delta function, which ensures that the term it multiplies has a non-negligible contribution to the integral. <sup>15</sup>

The indicator function in the first term of the expression means that equity holders receive a payout only when the firm is either in the liquid state, or dilutes equity prior to filing for Chapter

11. The next term in the formula is associated with distress costs. The third term represents the clearing of arrears when the unlevered firm value reaches  $V^B$  from below.<sup>16</sup> At this instant, if accumulated earnings exceed the arrears, the difference  $S_t - \theta A_t$  is positive and goes to the shareholders increasing the equity value. If the difference is negative, the firm has to issue additional equity to clear the arrears. We emphasize the dependence of the equity value on the default and liquidation times  $\tau_t^{VB}$  and  $\tau_t^{VL}$  respectively, because it will be useful in subsequent developments.

Note that if the firm is in default, the equity value depends on the path of EBIT, in particular on the time spent in bankruptcy and the value of  $S_t$ . This path dependence highlights the difference between our model and the extant literature. In particular, the computation of equity and debt values will be more involved.

The value of debt is equal to:

$$D(t, \delta_t, \tau_t^{VB}, \tau_t^{VL}) = \mathbf{E}^{\mathbb{Q}} \left\{ \int_t^{\tau_t} e^{-r(s-t)} \left[ c \mathbf{1}_{\{V_s \geq V^B\}} + \theta A_{s-} \delta^{\mathcal{D}}(s - \tau_t^{VB}) \right] ds \middle| \mathcal{F}_t \right\} + (1 - \alpha) \mathbf{E}^{\mathbb{Q}} \left\{ e^{-r(\tau_t - t)} (V_{\tau_t} + S_{\tau_t}) \middle| \mathcal{F}_t \right\}. \quad (11)$$

Note that creditors recover a fraction of earnings accumulated during the automatic stay at liquidation. Finally, the total value of the firm's assets is equal to:

$$\begin{aligned} v(t, \delta_t, \tau_t^{VB}, \tau_t^{VL}) &= E(t, \delta_t, \tau_t^{VB}, \tau_t^{VL}) + D(t, \delta_t, \tau_t^{VB}, \tau_t^{VL}) \\ &= \mathbf{E}^{\mathbb{Q}} \left\{ \int_t^{\tau_t} e^{-r(s-t)} \left[ \delta_s \mathbf{1}_{\{V_s \geq V^B\}} - \omega V_s \mathbf{1}_{\{V^L < V_s < V^B\}} + S_{s-} \delta^{\mathcal{D}}(s - \tau_t^{VB}) \right] ds \middle| \mathcal{F}_t \right\} \\ &\quad + (1 - \alpha) \mathbf{E}^{\mathbb{Q}} \left\{ e^{-r(\tau_t - t)} (V_{\tau_t} + S_{\tau_t}) \middle| \mathcal{F}_t \right\}. \end{aligned} \quad (12)$$

The fact that the EBIT accumulated during default stays in the firm is reflected in the third term in the second line of the expression.

We cannot evaluate the above expressions analytically. Instead we use the binomial lattice methodology developed in Broadie and Özgür Kaya (2005), an outline of which is given in the appendix.

### C. Optimal Debt and Equity Values

Up until now we have treated the value of  $V^B$  as fixed. Intuitively, this boundary should be located between the illiquidity ( $\delta_t = c$ ) and liquidation ( $\delta_t = \delta^L$ ) boundaries. However, it is not clear whether the firm should default immediately upon running out of earnings to pay the coupon, or should issue equity and default at or close to the boundary  $V^L$ . In order to answer this question, we first solve for  $V^B$  which maximizes the total firm value,  $v$ , subject to limited liability (i.e., the first best outcome). Then we compare the bankruptcy barrier obtained with the ones corresponding to the strategies which maximize equity value or debt value. As we will see, equity value maximization is not consistent with the first best outcome – a feature absent in the other models. In order to address this tension between the objectives of equity and debt holders, we investigate in Section IV how control can be shifted to achieve the first best outcome.

Given a value of  $V^B$  chosen above,  $V^L$  is determined endogenously by maximizing the firm value subject to the limited liability constraint.<sup>17</sup> As is the case with Leland (1994), the solution coincides with equity value maximization.<sup>18</sup> Overall, the liquidation region also depends on the length of the grace period,  $d$ , and on earnings accumulated in default  $S$ . In this respect our setting is different from Leland's where the liquidation region is constant.

More formally, we find the liquidation boundary via the stopping time:

$$\begin{aligned}\tau_t^{V^L} &= \arg \operatorname{ess\,sup}_{\tau \in \mathcal{L}} v(t, \delta_t, \tau_t^{V^B}, \tau) \\ \mathcal{L} &= \{\tau : \tau \geq \tau_t^{V^B}, E(t, \delta_t, \tau_t^{V^B}, \tau) > 0 \forall t < \tau\}\end{aligned}\tag{13}$$

We find the time of default in the first best case as:

$$\begin{aligned}\tau_t^{V^B} &= \arg \operatorname{ess\,sup}_{\tau \in \mathcal{B}} v(t, \delta_t, \tau, \tau_t^{V^L}) \\ \mathcal{B} &= \{\tau : E(t, \delta_t, \tau, \tau_t^{V^L}) > 0 \forall t < \tau\}\end{aligned}\tag{14}$$

Note that the liquidation time depends on the bankruptcy time (see (13)). Therefore, we find a new value of  $\tau_t^{V^L}$  for each trial value of  $\tau_t^{V^B}$  at the optimization stage in (14). Subsequently, we find the equity and debt value maximizing default boundaries using similar expressions.

We would like to make the following three observations regarding the optimization. First, the imposed constraints take into account the case when the arrears are cleared at  $V^B$  by issuing additional equity. If there is insufficient equity value then the firm is liquidated. Second, the determination of the optimal liquidation boundary  $V^L$  is similar to the determination of the optimal exercise boundary in American option pricing. The appendix briefly discusses how the optimization is implemented and further details are given in Broadie and Kaya (2005). Third, while the optimal  $V^B$  is constant due to time-homogeneity of the setup,  $V^L$  is path-dependent. This happens because  $\tau_t^{V^L}$  depends on the value of the suspended earnings account  $S_t$ .

For ease of comparison we first consider the case of a constant coupon level  $c$  across models. Then in Section V we explore the sensitivity of our analysis to the choice of  $c$ , discuss issues of debt capacity, and present results with optimal capital structure (i.e., optimal coupon level  $c$ ).

#### *D. Model of Bankruptcy Proceedings*

Our model is able to capture a number of salient features of bankruptcy proceedings. We explicitly incorporate the automatic stay provision by stopping all the payments to creditors, and keeping track of the accumulated interest and earnings. We also parsimoniously capture the spirit behind debtor-in-possession (DIP) financing. The grace period  $d$  does not exactly represent the exclusivity period – rather it is a length of time that firm is eventually allowed to spend in default, potentially after multiple renegotiations of the exclusivity period.<sup>19</sup> We allow for senior borrowing, which is typical for DIP financing, via the parameter  $\theta$ . This parameter serves a dual role. On the one hand it works as one of the distress costs to bondholders when it is not equal to zero. On the other hand,  $\theta$ 's proximity to zero shows how much arrears are going to be forgiven, which can be interpreted as additional (senior) borrowing.

An important part of the bankruptcy proceedings is who retains control at which stage. For example, Aghion and Bolton (1992) and Dewatripont and Tirole (1994) emphasize the importance of switching control from equity to debt in bad states. Moreover, Skeel (2003) points out that the recent trend in practice is that creditors are able to affect a lot of decisions in bankruptcy in contradiction to the letter of the bankruptcy law. This happens because creditors use DIP financing

and executive compensation as a lever to introduce new officers on the companies' boards and to provide incentive to executives to complete reorganization in a fast and efficient fashion. We are able to incorporate this features of the model by choosing both the bankruptcy level  $V^B$ , and the grace period  $d$  endogenously.

### *E. Related Structural Models*

Before we proceed with the discussion of the model properties and implications, we briefly discuss earlier papers which rely on structural models of default. In general, the development of this literature is driven by the desire to explain empirically observed credit spreads. The conventional wisdom is that structural models tend to underestimate credit spreads. This motivates more recent papers to introduce additional realistic features which would lead to an increase in spreads. Simultaneously, researchers attempt to preserve analytical valuation of debt and equity values. A model combining the two affects – analytical expressions and increased spreads – is considered to be successful.

Leland (2004) disputes whether this is an appropriate metric. He argues that default probability is a more informative measure than credit spreads for distinguishing different models. In practice spreads reflect not only the probability and severity of a credit event, but liquidity effects as well.<sup>20</sup> The extant structural models do not explicitly model liquidity effects, hence they are expected to fail in matching spreads.

Four recent papers that are close in spirit to the issues that we study here are by Francois and Morellec (2004), Galai, Raviv, and Wiener (2003), Moraux (2002), and Paseka (2003). These papers make a distinction between default and liquidation. Moreover they attempt to capture some dimensions of the bankruptcy code.<sup>21</sup>

The paper by Francois and Morellec (2004) keeps track of the cumulative time spent by the firm continuously under the default barrier. When this cumulative time exceeds the grace period  $d$ , the firm is liquidated. In their model the firm cannot be liquidated if the unlevered value of the assets is too low:  $\mathcal{T}_t = \tau_t^d \wedge \infty$ .<sup>22</sup> As a result, limited liability could be violated.<sup>23</sup> Absent limited liability, incentives of the creditors are much more closely aligned with those of debtors.

Therefore, it is not surprising that the authors limit themselves to finding the optimal values of  $V^B$  by maximizing the equity value only. As we will show, considering the first best and debt maximization scenarios leads to different implications for optimal bankruptcy decisions and securities values. In the Francois-Morellec model excursions below the default boundary are always associated with implicit forgiving of contractual obligations, i.e.,  $\theta = 0$ . Therefore, the authors find that the probability of liquidation decreases with  $d$ . Moreover, as we will see below, such an assumption can encourage equity to default even when  $\delta > c$ . Finally, the authors obtain closed-form solutions for the firm, debt, and equity values only prior to default.

Moraux (2002) attempts to penalize firms for default by keeping track of the cumulative time spent by the firm under the default barrier, which makes a difference for multiple bankruptcies. Apart from this feature, his setup is similar to that of Francois and Morellec. In particular,  $\theta$  is still equal to zero. He is able to find Laplace transforms of values of interest. Hence, the values themselves require numerical computation.

Galai, Raviv and Wiener (2003) recognize that Moraux's improvement ignores the severity of default, i.e., how far the unlevered firm value travels below  $V^B$ . In their model  $V^B$  is time-dependent and exogenous as in Black and Cox (1976). The firm liquidates when a new state variable, effectively computed as weighted average of the distances from  $V_t$  to  $V^B$  throughout the entire history of the firm's defaults, exceeds a certain value. Arrears in our model ( $\theta \neq 0$ ) capture the same effect in a more natural way that is consistent with actual bankruptcy practices. Galai, Raviv and Wiener derive partial differential equations and solve them numerically to obtain debt and equity values.

Paseka (2003), like our model, explicitly recognizes the presence of both default and liquidation boundaries. However, he specifies  $V^L$  as an exogenously determined fraction of  $V^B$ . This may lead to negative values of equity similar to the above papers. Paseka focuses on the outcome of the reorganization plan instead of the cash flows during the automatic stay period. The reorganization plan is proposed by equity as soon as  $V_t$  reaches a new boundary  $V^R > V^B$  and a dynamic bargaining game takes place. In equilibrium the plan is accepted, and the firm exits from bankruptcy debt free. Such a setup effectively gives all of the control to equity holders. As in the other papers, the first best and debt maximization scenarios are not explored. Finally, the author computes analytical

solutions for debt and equity as a function of debt value upon default,  $D(\delta^B)$ . However,  $D(\delta^B)$  and the optimal value of  $V^B$  require non-trivial numerical computations.

In summary, these papers make a contribution by considering the time spent in Chapter 11, the role of exclusivity period and the nature of excursions while in Chapter 11. However, none of the papers (with the exception of Paseka, 2003) considers two separate barriers for default and liquidation, and none of the papers considers the optimal choice of these two boundaries driven by different objectives. We believe that these issues are at the core of financial distress, because they allow us to assess the value the bankruptcy option. Endogenous  $V^B$  and  $V^L$  will tell us how early would a firm liquidate in the Leland (1994) world without bankruptcy as opposed to our model. This, in turn, will have implications for the values of debt, equity, and the firm overall.

## II. First Best Outcome

In this section we characterize first best outcomes, i.e., assuming that the total firm value is maximized, subject to limited liability, ex-ante upon filing for Chapter 11. This is clearly a useful theoretical benchmark. We then ask in a later section how first best outcomes can be enforced by lenders. Throughout, we will use Leland (1994) as the benchmark model corresponding to the case where only the liquidation option is available to the lenders and borrowers. This approach will help us to gain insight into the incremental value added by the bankruptcy code in alleviating liquidity problems. We will report all the quantities of interest, such the optimal default boundary  $V^B$ , or the firm value  $v(\delta_t)$ , as fractions of the related values implied by the Leland model.

In the discussions that follow, we have set the parameters of the model at the following values:  $\sigma = 20\%$ ,  $\mu = 1\%$ ,  $r = 5\%$ ,  $c = 3\%$ , and the initial conditions are:  $\delta_0 = 4$  (i.e.,  $V_0 = 100$ ),  $A_0 = 0$ , and  $S_0 = 0$ . We vary the costs of being under Chapter 11, denoted by  $\omega$ , from 0% to 2% and initially set the liquidation cost  $\alpha = 50\%$  (later we also consider extreme values of 10% and 90%).

<sup>24</sup> The debt forgiveness parameter  $\theta$  is set at either 100% (no forgiveness) or at 50% (half of the arrears forgiven). We vary the length of the grace period  $d$  that the firm is allowed to spend in Chapter 11 from zero to ten years. The choice of parameter values is motivated by the discussions in the calibration studies of Huang and Huang (2002) and Leland (2004).

## A. *Bankruptcy Boundary*

The first panel of the Figure 3 reports the bankruptcy boundary  $V^B$  as a fraction of the liquidation boundary  $V^L$  in the Leland (1994) model. The rationale is clear: in the Leland world  $V^B$  is equal to  $V^L$ . We would like to see if the availability of the Chapter 11 induces the firm to stop diluting equity earlier. One of the key implications from the plot is that the equity issuance region is always smaller once the Chapter 11 option is available to the agents.

[ Figure 3 is about here.]

Note that, from the perspective of firm value maximization, the illiquidity problem is sorted out sooner than in Leland (1994) by filing for Chapter 11. The particular choice of the bankruptcy level  $V^B$  will depend on the configurations of the debt forgiveness  $\theta$  and the distress cost  $\omega$ . Obviously, when there is no debt reduction ( $\theta = 100\%$ ), and no distress costs ( $\omega = 0\%$ ) the firm is willing to default early, especially if the grace period  $d$  is sufficiently large to minimize the probability of liquidation.

An interesting outcome of the model is that if there are distress costs ( $\omega = 2\%$ ) then the firm chooses to default earlier if some of the debt is forgiven. This happens because the liquidation cost  $\alpha$  forces the firm to delay default, i.e., moves closer to the Leland model, as in the case  $\theta = 100\%$  and  $\omega = 2\%$ . However, the ability to forgive part of the debt reduces the probability of the firm being liquidated, i.e., reduces the probability to incur liquidation costs, and therefore induces the firm to default earlier.

Since in our model the firm defaults earlier than in the Leland model under many circumstances, this implies that the probability of default is higher than in Leland (probability of default is determined by the default boundary, which is different across the models, and the properties of the EBIT generating process (1), which is identical for the different models). These probabilities are a big focus in research because many practitioners use Moody's|KMV EDF (expected default frequency), which is a closely related measure. Hence qualitative differences between our and extant models is potentially important. We will explore this issue in greater detail in Section B.

## *B. Firm, Equity, and Debt Values*

A natural first question is what happens to the firm value once the optimal, or first best, bankruptcy boundary is selected. To address this question, we plot in Figure 3 the total value of the firm as a fraction of the respective value in the Leland model. We see that the highest firm value is achieved with zero distress cost and partial debt forgiveness. This is natural because zero distress cost increases overall firm value (see equation (12)), and debt forgiveness increases the probability of avoiding costly liquidation. The firm values for the cases with either no forgiveness (and with no distress cost), or with distress cost (and with forgiveness) are smaller because one of the advantageous components is missing. Finally, the case with no debt forgiveness and with distress cost is naturally the worst, and basically coincides with the Leland benchmark.

As we have discovered, it makes sense for the firm as a whole to invoke Chapter 11 to get debt relief before liquidation if the Chapter 11 process is less expensive. Is it necessarily in the best interests of the lenders? We plot in Figure 3 the values of equity and debt. We see that both equity (debtors) and debt (creditors) benefit from the improvement in the firm value. Note, however, that Chapter 11 has the largest impact on bonds as they can be sold at a lower spread.

Since in many of the scenarios, the benefits of the code appear to pass to the lenders under the first best outcome, we must consider the two incentives implied by this result. First, lenders will have a strong incentive to move to the first best. Second, unless some concessions are given to the equity holders (in the form of debt forgiveness or reduced distress cost), they will not be interested in the first best outcome. Since the total value increases in the presence of the code, lenders will have the right incentives to make such concessions. These observations suggest that the presence of Chapter 11 as an additional outside option may lead to very interesting debt renegotiations. This is clearly an interesting topic for future research.<sup>25</sup>

We have experimented with other, more extreme, values of the liquidation cost. Qualitatively, the same results hold. Interestingly, when the cost is small ( $\alpha = 10\%$ ), liquidation is not a big threat to the firm. As a result, all the values converge to the one in Leland, i.e., the bankruptcy boundary is very close to the liquidation boundary despite the benefits of the Chapter 11. When the liquidation cost is high ( $\alpha = 90\%$ ), all the improvements observed in the base case of  $\alpha = 50\%$

are even stronger: Chapter 11 is beneficial for a firm with potentially costly liquidation. Last, but not least, in the presence of costly liquidation, a long grace period  $d$  increases the individual values of debt and equity, and the value of the firm as a whole. This happens precisely because it delays liquidation.

### **III. Default, Liquidation, and Values when Decisions are Made by Debtors or by Lenders**

#### *A. Debt, Equity and Firm Values*

So far we have analyzed the first best outcome. This is a natural theoretical benchmark for assessing how the outcomes might differ if borrowers or lenders have the ability to choose when to file for Chapter 11 or when to liquidate the firm. In the subsequent subsections, we will provide a discussion of these issues. We continue with the same parameter configurations that were used in the previous section.

As noted earlier, equity holders have a strong incentive not to follow the first best outcome as the results under such a scenario mostly favor lenders. Intuitively, in maximizing total value, we select the default boundary  $V^B$  lower than what equity holders might prefer: they would like to file for Chapter 11 sooner to get debt relief, especially when the grace period  $d$  is long. In fact, if the debt relief is high and the process of Chapter 11 is not too costly, they might file for Chapter 11 even when the EBIT is higher than promised coupon obligations, i.e., the firm is liquid. These observations are confirmed by Figure 4. Note that when the equity value is maximized, the debt value and the overall firm value decline relative to those in Leland (1994). This means that the equity holders have appropriated all the rents associated with the Chapter 11 option.

[ Figure 4 is about here.]

Clearly, the incentive of debtors to deviate from the first best outcome can be mitigated by restricting debt relief, as illustrated in Figure 4 (with  $\theta = 100\%$  and  $\omega = 2\%$ ). However, this may

not be total value maximizing, as noted earlier. Hence, as an alternative, sufficiently small values of  $d$  can keep the deviations from first best to be small. However, these remedies make the outside option of filing for Chapter 11 less valuable. A more productive approach is to let the lender take an active role in either deciding when the firm should file for Chapter 11 or taking the reins of the firm once the borrowers decide when to file for Chapter 11. Section IV is dedicated to this latter option. We consider the former option next.

Debt value maximization results are presented in Figure 5. <sup>26</sup> Comparing this to Figure 3, we immediately see that giving creditors full control leads to outcomes that are very similar to first best. When lenders are in control, the debt value is slightly higher and the equity value is slightly smaller. Despite being a useful alternative to equity maximization, debt maximization cannot be implemented in practice: typically equity holders make decisions about filing for Chapter 11.

[ Figure 5 is about here.]

However, as Skeel (2003) points out, in practice creditors are able to influence decision making in distress by introducing their officers into the firm's board and by using executive compensation contracts to align the incentives of managers with theirs. In the next section, we introduce a stylized way to transfer control in bad states.

### *B. Term Structure of Default Probabilities*

As we observed in the Section II, the presence of Chapter 11 increases the probability of default because the new default boundary is above that of Leland (1994). Indeed the probability that the firm defaults in  $t$  years,  $\mathbb{P}_B(t, \delta_0)$ , can be computed as:

$$\begin{aligned}
\mathbb{P}_B(t, \delta_0) &= \mathbb{P} \left( \sup_{0 \leq s \leq t} \delta_s \leq \delta^B | \delta_0 \right) \\
&= \Phi \left( \frac{-\log(\delta_0/\delta^B) - (\mu + \lambda - 0.5\sigma^2)t}{\sigma\sqrt{t}} \right) \\
&\quad + \exp \left( -\frac{2\log(\delta_0/\delta^B)(\mu + \lambda - 0.5\sigma^2)}{\sigma^2} \right) \Phi \left( \frac{-\log(\delta_0/\delta^B) + (\mu + \lambda - 0.5\sigma^2)t}{\sigma\sqrt{t}} \right)
\end{aligned} \tag{15}$$

where  $\mathbb{P}$  is the actual (as opposed to risk-neutral) probability measure,  $\lambda$  is the equity risk premium (associated with the probability measure switch from  $\mathbb{P}$  to  $\mathbb{Q}$ ), and  $\Phi(\cdot)$  is the standard normal cumulative distribution function (see Harrison, 1990 for details). In practice, one is interested in the actual, and not risk-neutral, probability of default, therefore we introduce the risk premium  $\lambda$ , which we take equal to 4%. In our model,  $\delta^B = V^B(r - \mu)$  (see (2)). In the case of Leland, default and liquidation boundaries coincide, and, therefore,  $\delta^B = V^L(r - \mu)$ .

While formal calibration of the model is beyond the scope of this paper (see Leland, 2004, for a relevant exercise), we contrast the term structure of default probabilities implied by our model and by Leland's model. In particular, we contrast the equity and debt maximization cases in our model. We use the firm's illiquidity boundary as a starting point ( $\delta_0 = c$ ), and compute the term structures of  $\mathbb{P}_B$  for the same parameter configurations. Figures 6 and 7 show the equity and debt maximization cases, respectively.

[ Figures 6 and 7 are about here.]

We see that, perhaps not surprisingly, the differences between the Leland model and our model are most dramatic when equity is in control. The only case with similar default term structures occurs when equity receives the harshest penalties for default (no debt forgiveness and the presence of distress costs). Generally, debt maximization aligns the default probabilities with Leland (1994) more closely. The exception is, naturally, when equity has to pay the arrears in full, and there are no distress costs, which causes lenders to default earlier.

Most importantly, irrespective of particular quantitative differences, the  $\mathbb{P}_B$ 's converge to zero, as  $t$  approaches zero, slower than in the case of the Leland model.<sup>27</sup> This happens because, as we saw, the default boundary is often higher than the default/liquidation boundary in the Leland case, which automatically leads to a higher probability of default according to equation (15). It appears that our model has sufficient flexibility to generate rich patterns of default probabilities. This property is a natural outcome of our attempt to distinguish default from liquidation.<sup>28</sup>

### C. *Default Versus Liquidation*

Our framework enables us to construct more precise measures of the differences between default and liquidation. For instance, one natural question to ask is whether the availability of Chapter 11 affects the chances of eventual liquidation. We can follow the strategy of the previous section and compute the probabilities of liquidation,  $\mathbb{P}_L$ , by replacing  $\delta^B$  by  $\delta^L$  in (15).

Similarly to the previous section, Figures 8 and 9 plot  $\mathbb{P}_L$  in the context of our and the Leland models for equity and debt maximization, respectively. The probability of liquidation depends on the grace period,  $d$ . Therefore, we restrict ourselves to the most interesting scenarios to avoid clutter. Based on the earlier discussions we set  $d = 2$ . A natural starting point for  $\delta_0$  is the level of EBIT after which scenarios with and without Chapter 11 diverge. This is the point where the firm files for Chapter 11, i.e.,  $\delta^B$ . Hence we are computing probabilities of liquidation starting at the default boundary.<sup>29</sup>

[ Figures 8 and 9 are about here.]

The situation is reversed as compared to the default probabilities. Our model implies lower probabilities of liquidation than those of Leland (1994). Hence, in many states of the world, the presence of Chapter 11 helps firms to avoid unnecessary liquidation. A longer grace period  $d$  would be helpful in reducing  $\mathbb{P}_L$ 's even further.

## IV. **Transfer of Control Rights**

As noted earlier, the intuition for ex-post transfer of control has been stressed in the incomplete contracting literature. An alternative to lenders taking over control of the decision to file for Chapter 11 is to give lenders the right of ex-post transfer of control once the decision is made by the borrowers to file for Chapter 11. While transfer of control can include many things, we consider a restricted version of transfer in which the lenders decide on the length of the grace period  $d$  once the firm is under Chapter 11. The choice of  $d$  plays off the potential liquidation threat (if  $d$  is

chosen to be too small) with the ability to sort out the illiquidity problem through automatic stay and debt relief.<sup>30</sup>

Specifically, in our model, creditors select the length of the grace period  $d$  at the time of default  $\tau_t^{V^B}$  by maximizing the debt value. The borrowers determine the default boundary  $V^B$  by maximizing the equity value taking into an account that creditors will be choosing  $d$  as described. This design is incentive compatible because the optimal grace period  $d$  that one computes ex-ante will be the same as the grace period selected at time  $\tau_t^{V^B}$ .

Figure 10 plots the bankruptcy boundary  $V^B$ , optimal value of  $d$ , and firm value as functions of  $\theta$ .<sup>31</sup> In addition to the previous parameter configurations, we explicitly report results for three values of the liquidation cost  $\alpha$ : 10%, 50%, and 90%. As before, when the liquidation cost is low, all values collapse to those in Leland (1994). For this reason, the case  $\alpha = 10\%$  is not shown to conserve space.

[ Figure 10 is about here.]

When the liquidation cost is high,  $\alpha = 90\%$ , and the forgiveness parameter  $\theta$  is below 0.8, the firm value is less than that implied by the Leland model. The threat of liquidation is very high for creditors because of the associated high costs. As a result, they are willing to give a long grace period to the debtors in the hope that they will eventually clear bankruptcy.<sup>32</sup> A long grace period removes the liquidation threat from the debtors, and they act as in the regular equity maximization case. The situation changes when  $\theta$  becomes sufficiently high (the amount of forgiven arrears declines). On the one hand, choosing a higher  $d$  leads to a greater chance that the firm may emerge solvent with some recovery on the arrears upon emerging from Chapter 11. On the other hand, a larger  $\theta$  encourages borrowers to choose a lower boundary for default (see Figure 4), and this erodes the residual value upon liquidation, for a given  $d$ , ceteris paribus. As long as there are no distress costs ( $\omega = 0\%$ ), the benefits of staying in Chapter 11 for a long period seem to outweigh the other costs, and a high  $d$  is chosen. When  $\omega$  is relatively high, lenders prefer to reduce  $d$ , and rely on the residual value upon liquidation to get their payout.

The most interesting interaction occurs for the intermediate value of  $\alpha$ . If the amount of forgiven arrears is small, the situation is the same as in the case of high liquidation costs: avoiding losses associated with waiting dominates liquidation costs, and, as a result, a small  $d$  is selected. When the debt forgiveness is substantial,  $\theta \leq 40\%$ , and the distress cost is small,  $\omega \leq 1\%$ , the discussed trade-offs reverse, but an optimal choice of  $d$  remains the same. Now, the lenders choose a small  $d$  to limit the size of maximum arrears that can be accumulated. In this case, choosing a small  $d$  has the effect of increasing the total firm value relative to the Leland case. Thus, even if the first best outcome is not achieved, there is movement in this direction.

## V. Optimal Coupon Rate

The ability of the firm to repay its debt affects the amount it can borrow, and, therefore, affects the optimal capital structure. In our context, the option of Chapter 11 should affect the ability of the firm to borrow, as pointed out in Acharya, Sundaram, and John (2004). It is easy to introduce tax advantages and identify the optimal level of debt. Following Goldstein, Ju, and Leland (2001), we apply corporate taxes to earnings adjusted for the interest payout, as opposed to reducing the interest payout in proportion to the tax rate. In particular, this removes the counterintuitive property that increases in taxes lead to increases in equity values. As a result, we have to scale the expression for equity value in (10) by  $(1 - \tau)$ . Here  $\tau$  represents the effective tax rate, which we assume to be equal to 15% (Leland, 2004 motivates this number). The expression for the firm value in (12) changes accordingly.

In this section we first discuss the optimal coupon value from the perspective of creditors. Next, we establish the coupon which optimizes the overall capital structure. Finally, we investigate how the optimal choice of debt affects other security values.

## A. Debt Capacity

We start by characterizing debt capacity. We plot in Figure 11 the value of the debt for each level of the promised coupon for the fixed grace period  $d = 2$  years. Note that the Chapter 11 option increases debt capacity because the firm is able to avoid premature liquidation.

[ Figure 11 is about here.]

## B. Capital Structure

Figure 12 illustrates the debt level which maximizes the overall firm value and reports the specific optimal coupon values  $c^*$  in the legend. Two interesting points are worth emphasizing. First, consistent with our previous observations, debt relief or small distress costs lead to the greater use of debt. Second, equity values are declining in the use of debt and basically coincide with the Leland values despite the different configurations of parameters controlling debt/bankruptcy relief.

33

[ Figure 12 is about here.]

## C. Optimal Firm, Equity and Debt Values

In this section we evaluate how the coupon selected to maximize the firm value affects the relationship between firm, equity and debt values in our model. Figure 13 reports the values of the bankruptcy boundary, firm value, equity value and credit spreads as a percentage of the respective values in the Leland (1994) model. However, in contrast to Figure 3, every point on the plots, corresponding to a different value of the grace period  $d$ , is computed based on a different, optimal, value of the coupon rate  $c$ . For consistency, we select both  $V^B$  and  $c$  to maximize the firm value.

[ Figure 13 is about here.]

We note that two of our main conclusions obtained for suboptimal coupon rate still hold. First, if Chapter 11 is available, a firm would stop diluting equity before the equity value reaches zero and would file for bankruptcy. Second, the firm value increases relative to the Leland model because the Chapter 11 proceedings are less costly than outright liquidation.

However, we notice the difference with the case where the coupon is assumed to be the same for all parameters: the bankruptcy boundary for the case  $\theta = 50\%$ ,  $\omega = 0\%$  is the highest, while it is one of the lower ones in Figure 3. This happens because the typical value of the optimal coupon (e.g.,  $c^* = 3.57\%$  when  $d = 2$ , see Figure 12) is not only larger than the constant value of three percent used before, but it is also much larger than optimal coupon values for other cases.

#### *D. Default and Liquidation*

As a final part of the optimal coupon analysis we report the default and liquidation probabilities along the lines of plots in Figures 6 – 9. We use the same parameter combinations, but consider the first best case, as opposed to equity or debt maximization. Figures 14 and 15 show results for default and liquidation, respectively.

[ Figures 15 and 14 is about here.]

Consistent with default probabilities in Figures 6 and 7, our model implies a higher probability of default than the Leland one. Therefore, this implication of our model is robust to the same level of debt across the models and to the optimal (and different) level of debt.

As in the earlier figures, the probability of liquidation is lower in our model. However, now the difference between our and Leland's model is minimal (one to two basis points). One has to be careful interpreting these results since, as in Figure 13, we are comparing firms with different leverage. One interpretation is that, given the option of Chapter 11, a firm can raise more debt without increasing the liquidation risk.

## VI. Conclusion

We have presented a stylized model of a firm which has risky debt outstanding in its capital structure. The lender and the borrower have the options of filing for Chapter 11 and liquidating (Chapter 7). Chapter 11 in our model takes into account automatic stay, grace period and debt relief. The first best outcome is shown to be different from the outcome when equity chooses the value at which to enter Chapter 11. In this case equity holders will be able to appropriate value ex-post at the expense of debt holders by filing early for Chapter 11. This result is in sharp contrast to Leland (1994) who shows that, with the liquidation option only, the first best outcome coincides with the equity value maximizing outcome. In our numerical results, the first best outcome could be restored, in large part, by giving creditors the right to select the length of the grace period once the firm is taken to Chapter 11 by the equity holders.

Irrespective of who is in control, our model generates probabilities of default that are larger than those in the Leland model. In particular, the probabilities converge to zero slower as maturities decline. Interestingly, our model often generates lower probabilities of liquidation as compared to Leland. This implies that Chapter 11 facilitates recovery of firms from financial distress.

## Appendix: Computational Methodology

In this appendix we outline the computational methodology used to solve the model and generate the results described in the paper. A more detailed treatment is given in the paper Broadie and Kaya (2005), which also describes numerical procedures for solving the models of Leland (1994), Francois and Morellec (2004), and others. One advantage of a numerical approach is the ability to incorporate finite maturity debt, which leads to time-varying liquidation boundaries. Previous analytical solutions relied on constant liquidation boundaries which arise with infinite maturity debt. Another advantage of a numerical approach is the ability to incorporate additional model complexity, e.g., automatic stay provisions, arrears payments, grace periods, etc. These model features typically introduce path dependencies which make analytical solutions difficult, if not impossible. Path dependencies are handled in a backwards solution procedure by increasing the state space to include variables which record the values of path dependent quantities. Also, a numerical optimization procedure is generally required to determine optimal parameters in debt value, equity value, or firm value maximization.

The computational approach is based on a discrete approximation to the unlevered asset value process of the firm. Define  $\gamma = r - \mu$ , then equation (2) gives

$$\frac{dV_t}{V_t} = (r - \gamma)dt + \sigma dW_t(\mathbb{Q}). \quad (16)$$

This is the same representation as the Black-Scholes model for a stock price that pays dividends at a constant rate  $\gamma$ . Equation (16) can be discretized in many ways. For illustration, we'll use the standard binomial lattice approximation of Cox, Ross, and Rubinstein (1979). Starting from an asset value of  $V$  at time  $i\Delta t$ , the asset value will be  $V_h = hV$  with probability  $p$  or  $V_l = lV$  with probability  $1 - q$  at time  $(i + 1)\Delta t$ . The upmove multiple  $h$  is  $e^{\sigma\sqrt{\Delta t}}$ , the downmove multiple is  $l = 1/h$ , and the risk-neutral probability of an upmove is  $q = (b - l)/(h - l)$ , where  $b = e^{(r - \gamma)\Delta t}$ . At each node  $V$  additional information will be recorded, including the debt value  $D$ , the equity value  $E$ , the firm value  $v$ , and other state variables as needed.

A generic binomial step involves moving from known equity, debt, and firm values at time  $(k + 1)\Delta t$  to their respective values at the previous time  $k\Delta t$ , e.g.,

$$\begin{aligned} E &= e^{-r\Delta t} [qE_h + (1 - q)E_l] \\ D &= e^{-r\Delta t} [qD_h + (1 - q)D_l] \\ v &= e^{-r\Delta t} [qv_h + (1 - q)v_l]. \end{aligned} \quad (17)$$

These generic equations need to be modified to account for specific model features, e.g., coupon payments, limited liability of equity, etc. To illustrate, suppose the debt coupon payment at each time  $0, \Delta t, 2\Delta t, \dots, n\Delta t$  is  $c\Delta t$  on a bond with a face value of  $P$  which matures at finite time  $T = n\Delta t$ .<sup>34</sup> Further suppose that the cash flow (dividends) produced by the assets of the firm in each time period of length  $\Delta t$  is  $V_t(e^{\gamma\delta_t} - 1)$ . By a slight abuse of notation, we'll call this value  $\delta_t$ . At the bond maturity, the cash flows and the debt, equity and firm values are known. Thus for all nodes  $V_T$  at time  $T$  we have:

$$\begin{aligned} \text{If } V_T + \delta_T \geq c\Delta t + P: \quad E &= V_T + \delta_T - c\Delta t - P \\ D &= c\Delta t + P \\ v &= V_T + \delta_T \\ \text{If } V_T + \delta_T < c\Delta t + P: \quad E &= 0 \\ D &= (1 - \alpha)(V_T + \delta_T) \\ v &= (1 - \alpha)(V_T + \delta_T). \end{aligned} \quad (18)$$

In the first case, the firm is solvent at maturity; in the second case the firm is liquidated, and a liquidation cost of  $\alpha(V_T + \delta_t)$  is incurred.

At nodes prior to time  $T$ , equity, debt, and firm values are determined. We first illustrate when limited liability is considered, but Chapter 11 features are ignored. Since liquidation is determined by the value of equity, we first compute the present value of equity ignoring any liquidation event,  $\tilde{E}$ :

$$\tilde{E} = e^{-r\Delta t} [qE_h + (1 - q)E_l]. \quad (19)$$

No liquidation occurs if the present value of equity  $\tilde{E}$  plus current dividends are sufficient to make the bond coupon payment; otherwise liquidation occurs. Hence, the basic recursion is modified to:

$$\begin{aligned}
\text{If } \tilde{E} + \delta_t \geq c\Delta t : \quad E &= \tilde{E} + \delta_t - c\Delta t \\
D &= c\Delta t + e^{-r\Delta t} [qD_h + (1-q)D_l] \\
v &= \delta_t + e^{-r\Delta t} [qv_h + (1-q)v_l] \\
\text{If } \tilde{E} + \delta_t < c\Delta t : \quad E &= 0 \\
D &= (1-\alpha)(V_t + \delta_t) \\
v &= (1-\alpha)(V_t + \delta_t). \tag{20}
\end{aligned}$$

This recursion is continued working backwards until time zero.

Note that the equity value can be written as  $\max(\tilde{E} + \delta_t - c\Delta t, 0)$ , i.e., the maximum of continuation and stopping values, as seen from the perspective of equity holders. The equity value is seen as the value of an optimal stopping problem and the smooth pasting condition will be obtained in the limit as  $\Delta t$  goes to zero. See Dixit and Pindyck (1994, pp.130-132) for an elementary derivation of the smooth pasting condition in optimal stopping problems. Leland (1994) uses the smooth pasting condition to derive a formula for the optimal liquidation boundary in his model.

The binomial approach offers an alternative proof that the liquidation boundary which maximizes firm value subject to the limited liability constraint also maximizes equity value. To see this, consider the terminal time  $T$  in the case  $V_T + \delta_T < c\Delta t + P$  (see equation (18)). If the firm is liquidated, then  $v = (1-\alpha)(V_T + \delta_T)$  and  $E = 0$ . However, if the firm is not liquidated then  $v = V_T + \delta_T$  and  $E = V_T + \delta_T - c\Delta t - P < 0$ . The equity value can be written as the maximum of the continuation value  $(V_T + \delta_T - c\Delta t - P)$  and the stopping value (0). The firm value can trivially be written as the maximum of the continuation value  $(V_T + \delta_T)$  and the stopping value  $(1-\alpha)(V_T + \delta_T)$  subject to the limited liability constraint. In other words, the action which maximizes equity value also maximizes firm value subject to the limited liability of equity constraint. A similar statement holds at time  $T$  in the case  $V_T + \delta_T \geq c\Delta t + P$ .

Now consider any earlier time  $t$  and proceed by induction. In the case  $\tilde{E} + \delta_t \geq c\Delta t$  (see equation (20)), if the firm is liquidated, then  $v = (1-\alpha)(V_t + \delta_t)$  and  $E = 0$ . If the firm is not liquidated, then  $v = V_t + \delta_t$  and  $E = \tilde{E} + \delta_t - c\Delta t > 0$ . The equity value can be written as the maximum of

the continuation value (a positive value) and the stopping value (0), where the continuation decision maximizes equity value. In this case, the continuation decision also maximizes firm value subject to the limited liability constraint. Similarly, at time  $t < T$  in the case  $\tilde{E} + \delta_t < c\Delta t$ , if the firm is not liquidated, then  $v = V_t + \delta_t$  and  $E = \tilde{E} + \delta_t - c\Delta t < 0$ . If the firm is liquidated, then  $v = (1 - \alpha)(V_t + \delta_t)$  and  $E = 0$ . The equity value can be written as the maximum of the continuation value (a negative value) and the stopping value (0), where the stopping decision maximizes equity value. In this case, the stopping decision also maximizes firm value subject to the limited liability constraint.

So in the binomial model, at all times in all states, the action which maximizes equity value also maximizes firm value subject to the limited liability of equity constraint. The same result holds when Chapter 11 proceedings and distress costs are included in the model (apply the same arguments as above to the equations in Section 5 of Broadie and Kaya 2005).

The liquidation boundary (i.e., nodes where the transition from  $\tilde{E} + \delta_t \geq c\Delta t$  to  $\tilde{E} + \delta_t < c\Delta t$  occurs) is determined endogenously by this procedure. For finite maturity bonds the liquidation boundary will be time dependent. This numerical procedure can also handle discrete coupon payments and multiple debt instruments.

When modeling Chapter 11 proceedings, additional information needs to be recorded at each node in the binomial lattice to track (i) the coupon payments accumulated in the arrears account, (ii) the cash accumulated in the suspended earnings account, and (iii) the time spent in bankruptcy. However, with constant coupon payments, knowledge of the time spent in bankruptcy is enough to deduce the coupon payments accumulated in the arrears account. Thus only two additional state variables are required.

The state variable  $\ell$  is used to determine the firm's location on the grid over the time spent in bankruptcy:  $[0, \Delta t, 2\Delta t, \dots, m\Delta t]$ . For simplicity we assume that  $m$  is an integer, but it is not difficult to relax this assumption. The grace period expires when  $m\Delta t = d$  and the firm is liquidated. The state variable  $S$  is used to track the suspended earnings. In practice, it is discretized and can take values in  $[0, \bar{S}/M, 2\bar{S}/M, \dots, \bar{S}]$ , where  $\bar{S} = V^B(e^{\gamma\Delta t} - 1)m\Delta t$  is an upper bound on the value that  $S$  can take and  $M$  is an integer which determines the size of the  $S$  grid.

At an asset value node  $V$  at time  $k\Delta t$ , the equity value of the firm depends not only on  $V$ , but also on  $\ell$  and  $S$ . Suppose  $\ell = i\Delta t$  and  $S = j\bar{S}/M$ . In this case, we denote the equity value by  $E[i, j]$ . Similarly the debt and firm values are denoted by  $D[i, j]$  and  $v[i, j]$ , respectively. The recursions developed earlier are modified to take into account the current state  $(V, i, j)$  and the possible transitions at time  $(k+1)\Delta t$ . In the next period,  $\ell$  can change to either  $(i+1)\Delta t$  (remain in bankruptcy) or 0 (bankruptcy is cleared), where both are exact grid values. However,  $S$  can change from  $j\bar{S}/M$  to values that are not integer multiples of  $\bar{S}/M$  (i.e., are not contained in the grid of  $S$  values), and interpolation is required.

When Chapter 11 proceedings are included in the model, liquidation may occur when the time spent in default exceeds the grace period. When this constraint is binding, the smooth pasting condition will not hold, even in the limit as  $\Delta t$  goes to zero. However, the numerical procedure can be used to determine the (path dependent) liquidation region.

There are many cases to consider in developing the recursions. For example, one case is that the firm is currently above the bankruptcy boundary and makes a transition into the bankruptcy region. Another case is that the firm is in bankruptcy and stays in bankruptcy the next period. The details of the recursions, example computations, and convergence results are given in Broadie and Kaya (2005). This paper also discusses the optimization procedure used to determine the  $V^B$  which maximizes debt, equity, or firm values.

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## Endnotes

1. Our paper will have nothing to say about the optimal choice between the different workout alternatives to bankruptcy proceedings. Nevertheless, our results will provide the benchmark for making this decision.
2. The grace period is the time that creditors give the debtors to recover from distress before the firm is liquidated. The grace period could be thought of as a renegotiated exclusivity period – the time frame within which a reorganization plan must be filed.
3. In this paper we do not distinguish equity holders from management, so both parties are referred to as “debtor,” or “borrower,” or simply “equity.” Likewise, we will use “creditor,” “lender,” and “debt” interchangeably.
4. Such strategic debt servicing possibilities have been explored by Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997).
5. Rational lenders will anticipate this action by borrowers and accordingly charge a higher coupon rate.
6. Note that Leland (1994) does not necessarily require liquidation at  $V^L$ . If the bankruptcy costs driven by  $\alpha$  are higher than the restructuring costs, it might be advantageous to renegotiate the unprotected debt on the brink of bankruptcy. We distinguish the bankruptcy state from the liquidation state, a distinction not present in the Leland (1994) model. Our variable  $V^L$  corresponds the variable  $V_B$  in Leland’s (1994) notation.
7. Although, once under bankruptcy there are some real costs that are borne by the firm.
8. Hackbarth, Hennessy, and Leland (2003) consider an optimal mixture of bank and public debt. Fan and Sundaresan (2000) study the exchange offers. Chen (2003) determines the optimal choice of workout vs bankruptcy in the presence of asymmetric information.

9. Guha (2002) provides detailed analysis of these indentures and their impact on recovery.

10. In practice, unsecured creditors are rarely eligible for the accrued interest. We do not model secured credit explicitly, however the limited period allowed to stay in bankruptcy implicitly imposes an upper bound (equal to  $\omega V^B d$  – see the notation and the computations below) on the resources that could be wasted as a result of bankruptcy. This feature automatically secures a fraction of the firm’s resources.

11. The assumption that the firm clears default at the same level it filed for bankruptcy is our modeling choice. We do not see how to consider an endogenous bankruptcy-clearing level in a more meaningful way. If endogeneity were the case, the optimal boundary would change each instant as a function of the current EBIT, time in bankruptcy and distance to liquidation. In particular, it would be in the equity holders’ best interest to lower the boundary right before the expiration of a grace period.

12. Emanuel (1983) models the arrears on the preferred stock dividend in a similar fashion. However, since his focus is on the valuation of preferred stock, there is no role for debt in his model. Bartolini and Dixit (1991) value sovereign debt, and hence do not allow for default. They model the arrears by letting the sovereign borrower capitalize the unpaid interest.

13. If there is not sufficient equity, the equity value will become negative. In Section 2.3 the limited liability requirement is imposed, which implies that the firm will be liquidated if there is not sufficient equity remaining to repay the arrears.

14. On average, Chapter 11 cases last 2.5 years (see Helwege, 1999 and the references therein).

15. The Dirac delta function has the following properties:  $\delta^D(x - x_0) = 0$ , if  $x \neq x_0$ , and  $\int \delta^D(x - x_0)f(x)dx = f(x_0)$ . For example, as a result of being multiplied by the Dirac delta function, the integral of the arrears  $A_s$  over a range of values of  $s$  will be equal to  $A_{\tau_t^{VB}}$  if the range includes  $\tau_t^{VB}$ , or to zero otherwise.

16. Note that the Dirac function value does not distinguish whether the bankruptcy boundary is reached from above or below. However, if  $V^B$  is reached from above, both  $A_{t-}$  and  $S_{t-}$  will be equal to zero (see (4) and (5)).

17. This assumption is consistent with the bankruptcy court which determines liquidation based on the socially optimal considerations (i.e., not liquidating a firm when there is positive firm value remaining).

18. This follows from the argument given in the appendix following equation (20).

19. In Section IV we explicitly address negotiation over  $d$ .

20. Here the notion of liquidity is different from the one we use throughout the paper. In the present context, liquidity refers to the ease of trading in the particular bond.

21. In addition, Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) have analyzed strategic debt servicing behavior of borrowers under costly liquidation.

22. In other words, while Leland (1994) allows only for  $V^L$ , Francois and Morellec (2004) allow only for  $V^B$ .

23. The authors address this problem by introducing bargaining, which splits up the firm value, which is non-negative, between equity and debt at the liquidation state.

24. Note that in the Leland (1994) model the value of  $\alpha$  that we use implies default costs, which depend on the value of  $V^L$ , of 65%. This number is much higher than the ones used in the calibration literature (50%) and reported by the empirical studies, such as Andrade and Kaplan (1998) (10%-20%). In our model the liquidation level depends on the grace period  $d$  and will be lower than that in the Leland model in many scenarios. We might still overstate the cost of default in our examples. It means that the benefits of Chapter 11 will be lower in these cases.

25. For example, see Anderson and Sundaresan (1996), and Mella-Barral and Perraudin (1997) for models of debt renegotiation.

26. We assume that debt controls the choice of  $V^B$  while  $V^L$  is still effectively determined by the limited liability of equity restriction.

27. There are only two models which rely on alternative mechanisms to generate deviations from zero at any maturity. Zhou (2001) adds a jump component to the unlevered firm value  $V$ . Duffie and Lando (2001) introduce default uncertainty at short horizons via additional noise term (representing imprecise accounting information) in the dynamics of  $V$ . As a result, there is a non-zero probability of default in these models even as the horizon  $t$  in (15) approaches zero.

28. Note that other mechanisms, such as continuous finite-term debt rollover in Leland and Toft (1996) can generate a similar effect.

29. This level has no particular meaning in the Leland (1994) model.

30. Empirical evidence in Helwege (1999) emphasizes the link between the length of default and bargaining.

31. We restrict the range of possible values of  $d$  to the interval  $[0, 10]$  due to the computational burdens. This restriction occasionally binds as can be seen in the figure. We comment on this later where appropriate.

32. It is clear from the bottom right panel of the figure that our implicit restriction on  $d$  binds for low values of  $\theta$  and for the case  $\omega = 0\%$ . When  $\omega = 0\%$  it is costless for creditors to extend  $d$  indefinitely in order to avoid liquidation costs.

33. Because of such similarity with the Leland results, we do not provide the respective plots to conserve space. The results are available upon request, or could be deduced from the equation (17)

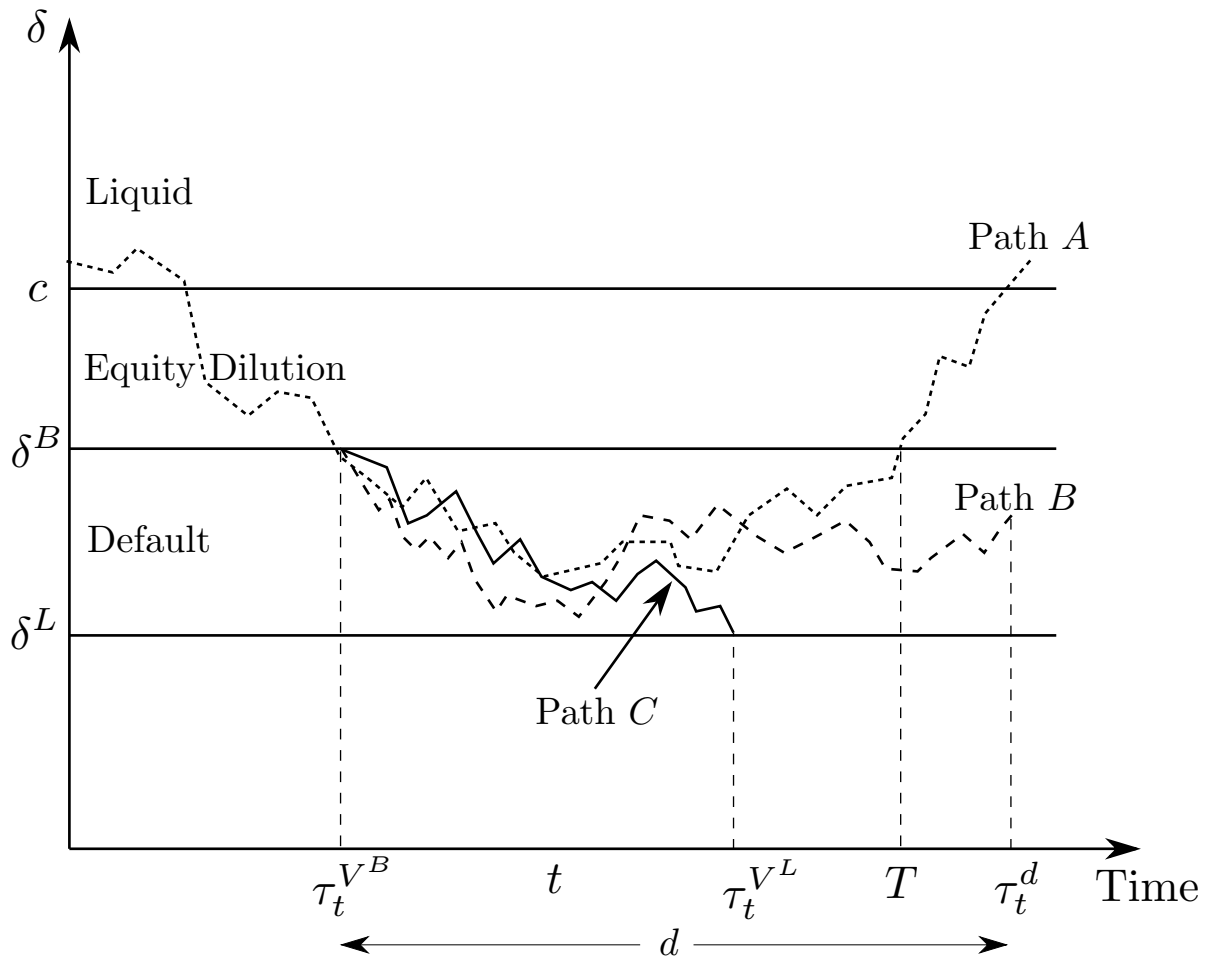
in Leland (1994).

34. A consol (infinite maturity) bond is approximated by taking  $T$  large and modifying the boundary conditions in equation (18) accordingly. In our computations we take  $T = 200$  and check that the results do not differ appreciably for even larger values of  $T$ .

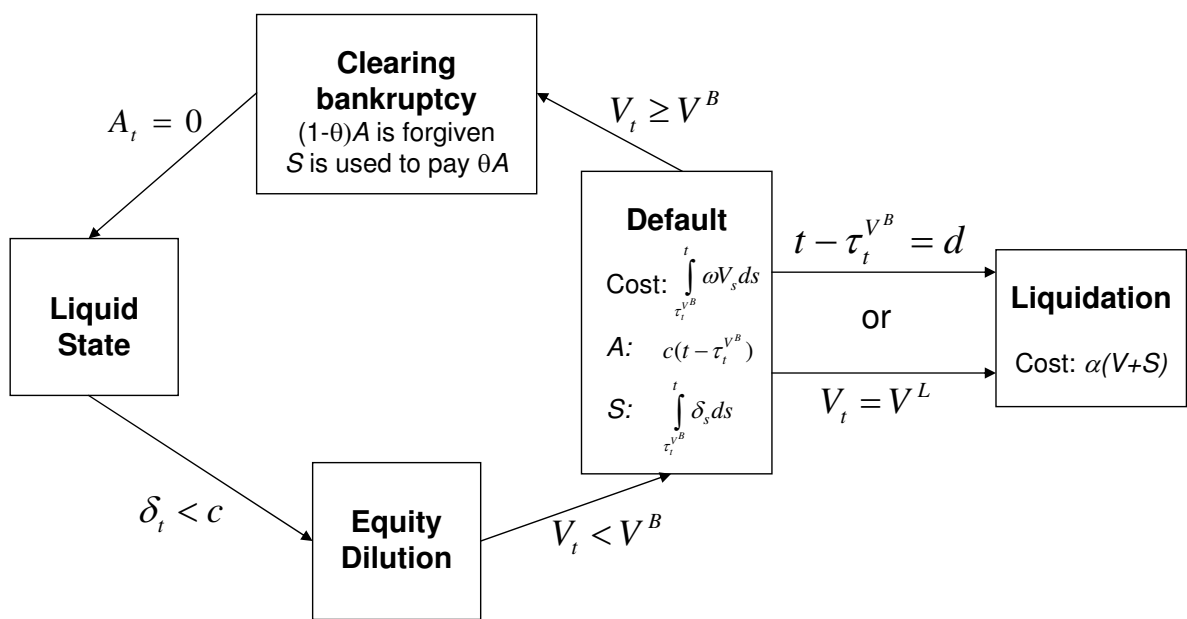
**Table I**  
**Notation**

The table summarizes the parameters used in the model and provides the base values for the numerical examples.

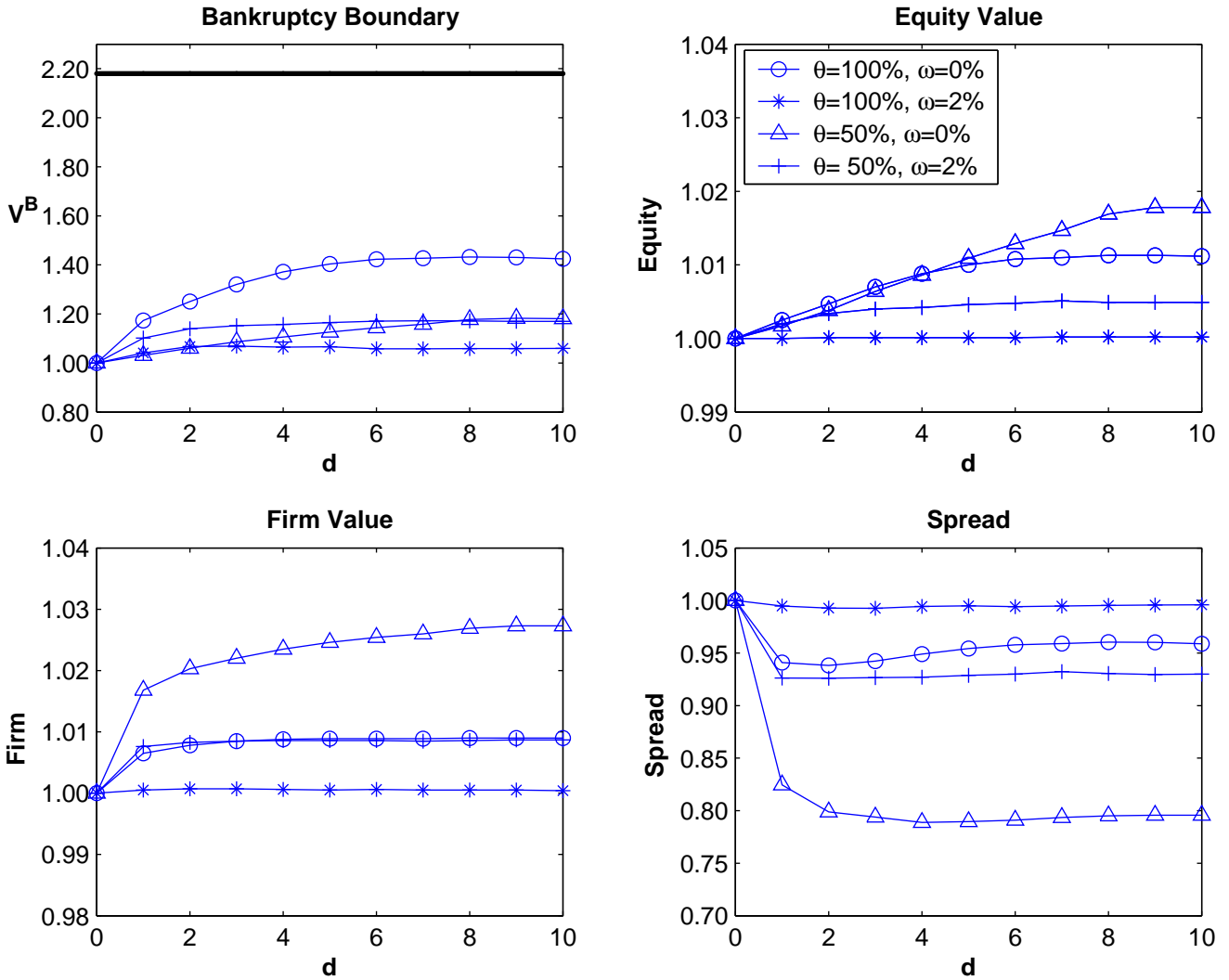
Notation	Explanation	Base value
<b>The setup</b>		
$\delta_t$	EBIT (payout flow)	
$\mu$	Drift of EBIT under $\mathbb{Q}$	1%
$\sigma$	Volatility of EBIT	20%
$r$	Risk-free interest rate	5%
$c$	Coupon rate	3%
$V_t$	Present value of EBIT (Value of unlevered assets)	
$E(\delta_t)$	Equity value	
$D(\delta_t)$	Debt value	
$v(\delta_t)$	Firm value	
<b>Default</b>		
$V^B$	Default boundary	
$d$	The grace period – maximal amount of time the firm is allowed to stay in default	0 – 10 years
$\tau_t^{V^B}$	the last time the firm's value reached $V^B$ prior to $t$	
$\tau_t^d$	time of liquidation due to long time spent in default ( $t - \tau_t^{V^B} \geq d$ )	
$\tau_t^{V^L}$	time of liquidation due to limited liability violation	
$A_t$	Arrears	
$\theta$	fraction of the arrears to be paid out	50% or 100%
$S_t$	the amount of earnings accumulated in the default	
$\omega$	Distress costs	0% – 2%
$\mathbb{P}_B(t, \delta_0)$	Probability of default in $t$ years starting at $\delta_0$	
<b>Liquidation</b>		
$V^L$	Liquidation boundary	
$\alpha$	Liquidation costs	50%
$\mathbb{P}_L(t, \delta_0)$	Probability of liquidation in $t$ years starting at $\delta_0$	



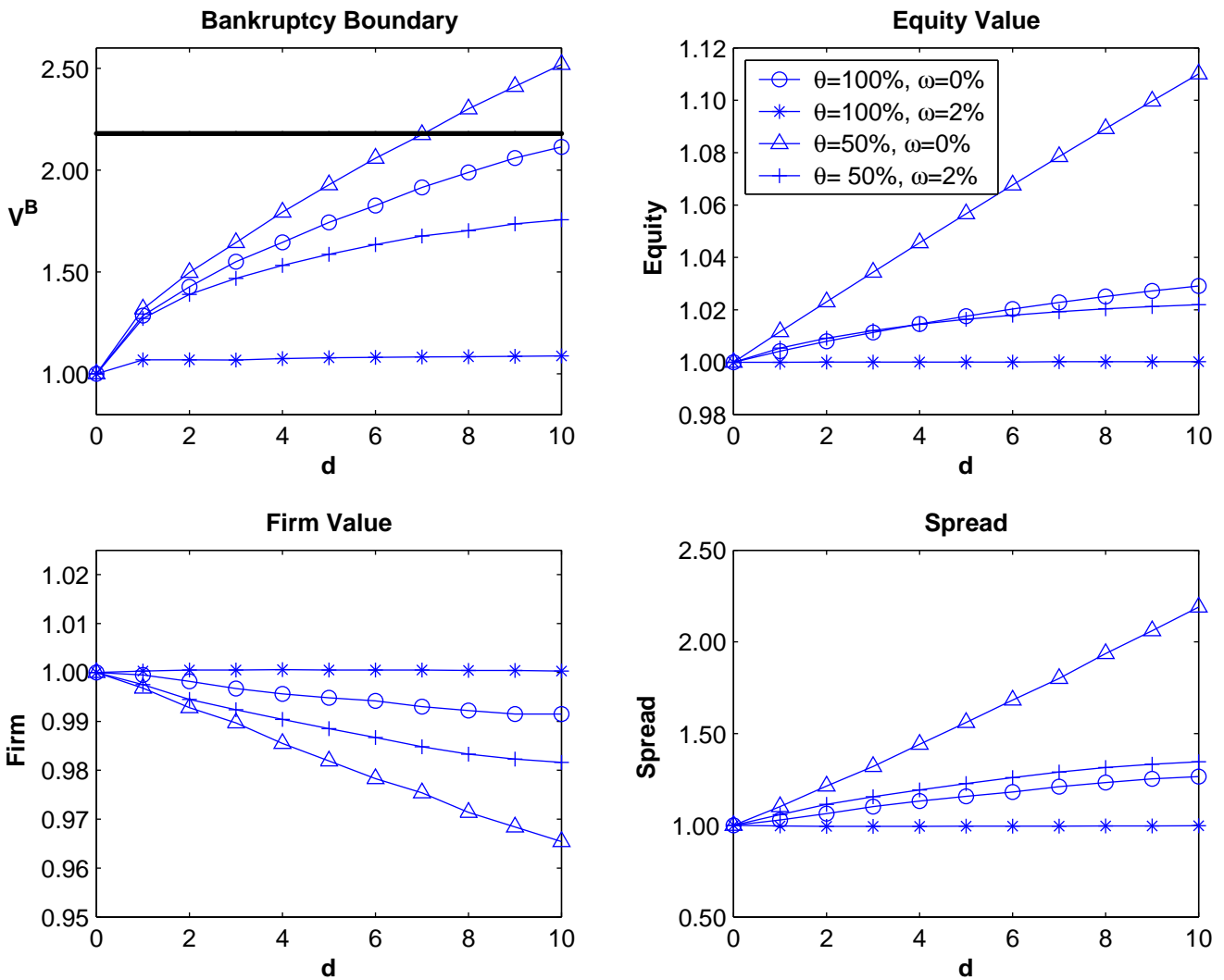
**Figure 1. Time Series of Events.** In this figure we plot possible paths of the EBIT process ( $\delta_t$ ) and the key boundaries which determine the state of the firm:  $\delta_t = c$  (distress),  $\delta^B$  (Chapter 11) and  $\delta^L$  (liquidation, or Chapter 7). There are three principal scenarios that are possible after the firm defaults by crossing the bankruptcy boundary  $\delta^B$  at time  $\tau_t^{V^B}$ . Path A corresponds to a “successful” bankruptcy, i.e., at a time  $T$  the firm is able to clear the default by paying out a fraction of arrears  $\theta A_T$  to the creditors by using the EBIT accumulated in the account  $S$ , and by diluting the equity value if necessary ( $S_T < \theta A_T$ ). Path B corresponds to the default leading to the liquidation because of the firm’s overstay in bankruptcy. Finally, path C corresponds to the liquidation due to the equity value reaching zero.



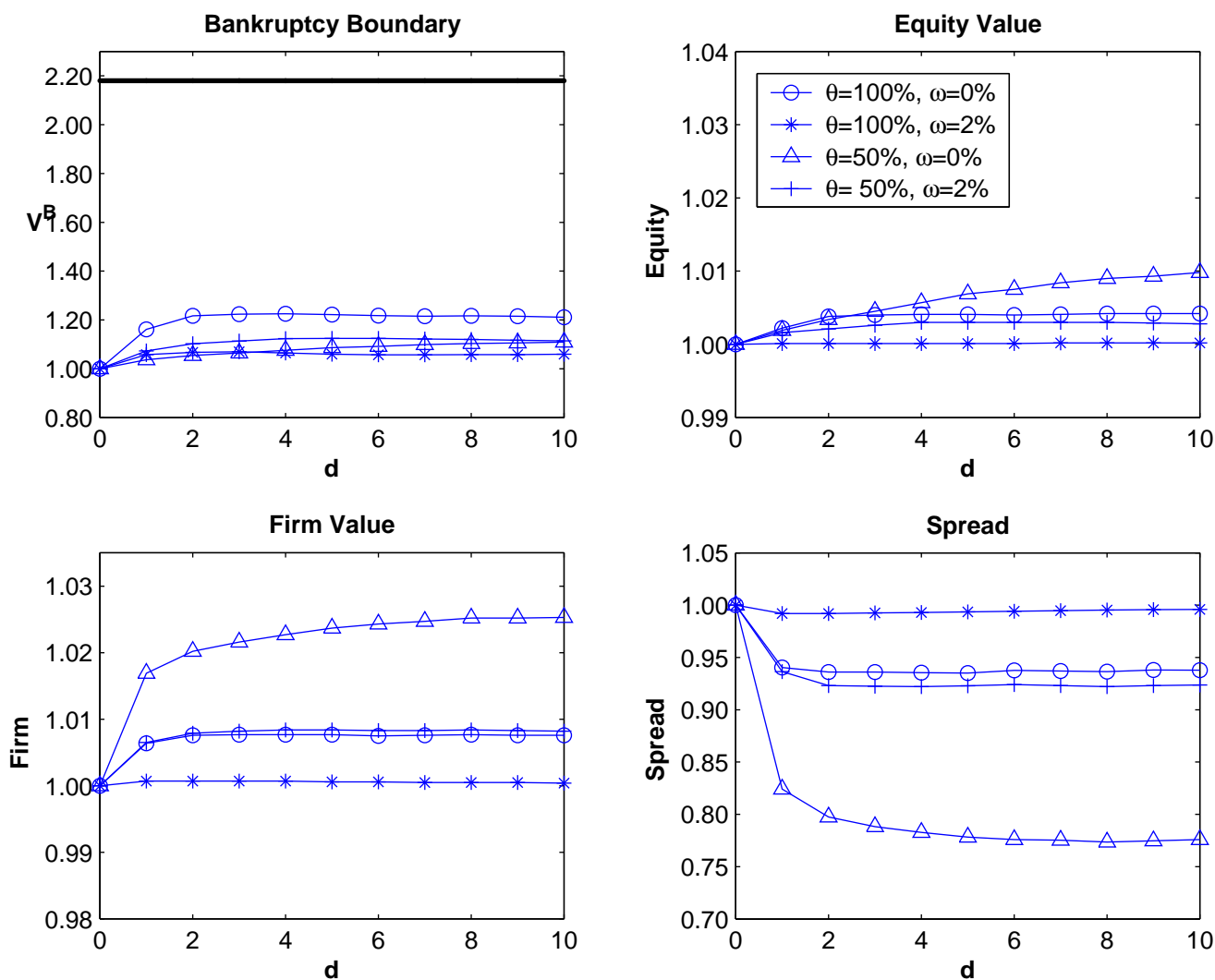
**Figure 2. Sequence of Events.** The flowchart shows the events that can occur during a firm's lifespan. Please refer to Table I for a summary of the notation. The labels around arrows indicate current conditions under which the firm switches into a new state. The firm goes into the 'Liquidation' state when one of the two indicated conditions is satisfied.



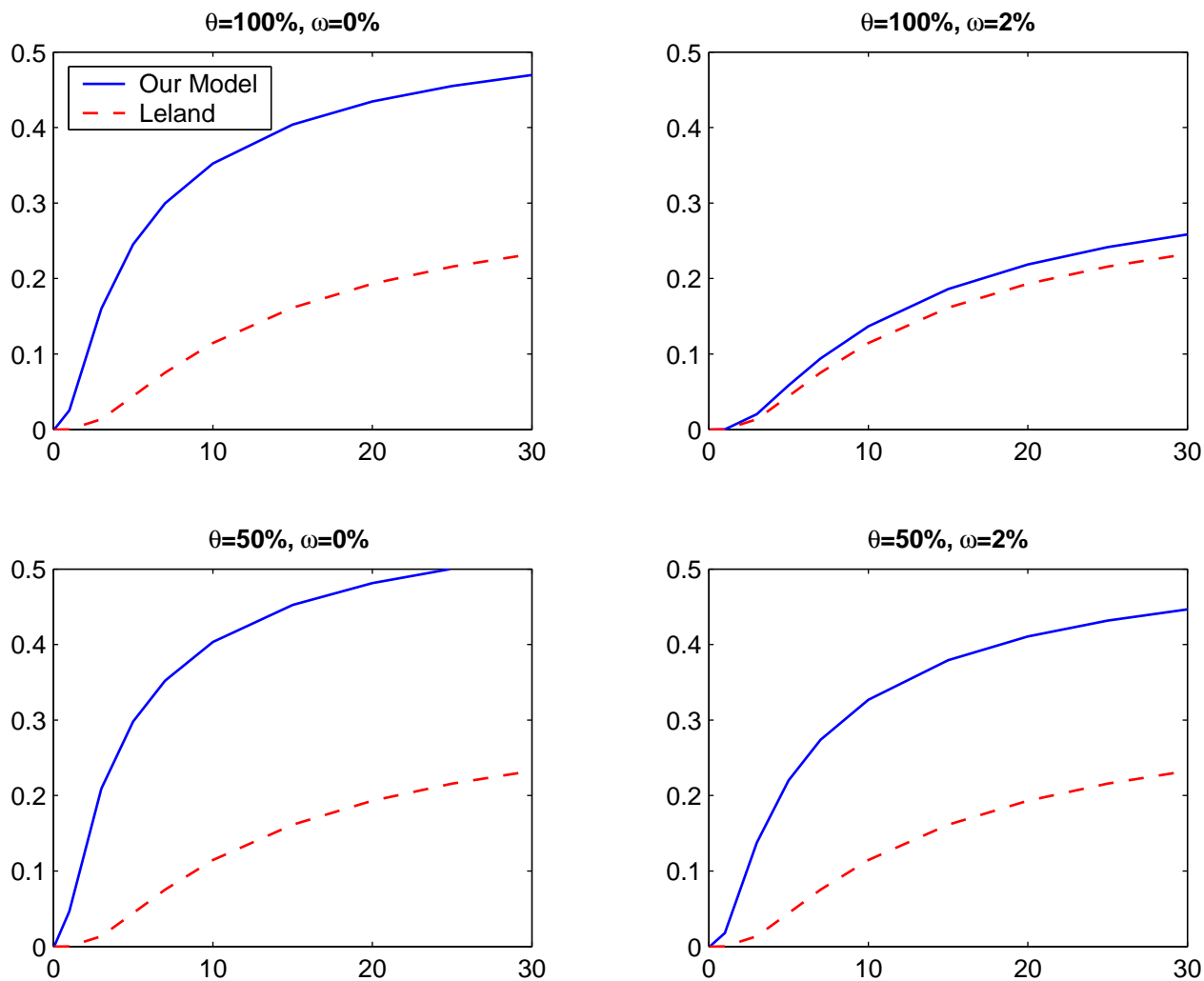
**Figure 3. Firm Value Maximization (First Best).** These plots correspond to the case when the bankruptcy boundary is selected to maximize the overall firm value (first best). The panels show the optimal bankruptcy boundary, firm and equity values, and the credit spread as a percentage of the benchmark Leland (1994) model for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ . The values of other parameters are provided in Table I. The straight line in the upper left plot designates the illiquidity boundary ( $\delta_t = c$ ).



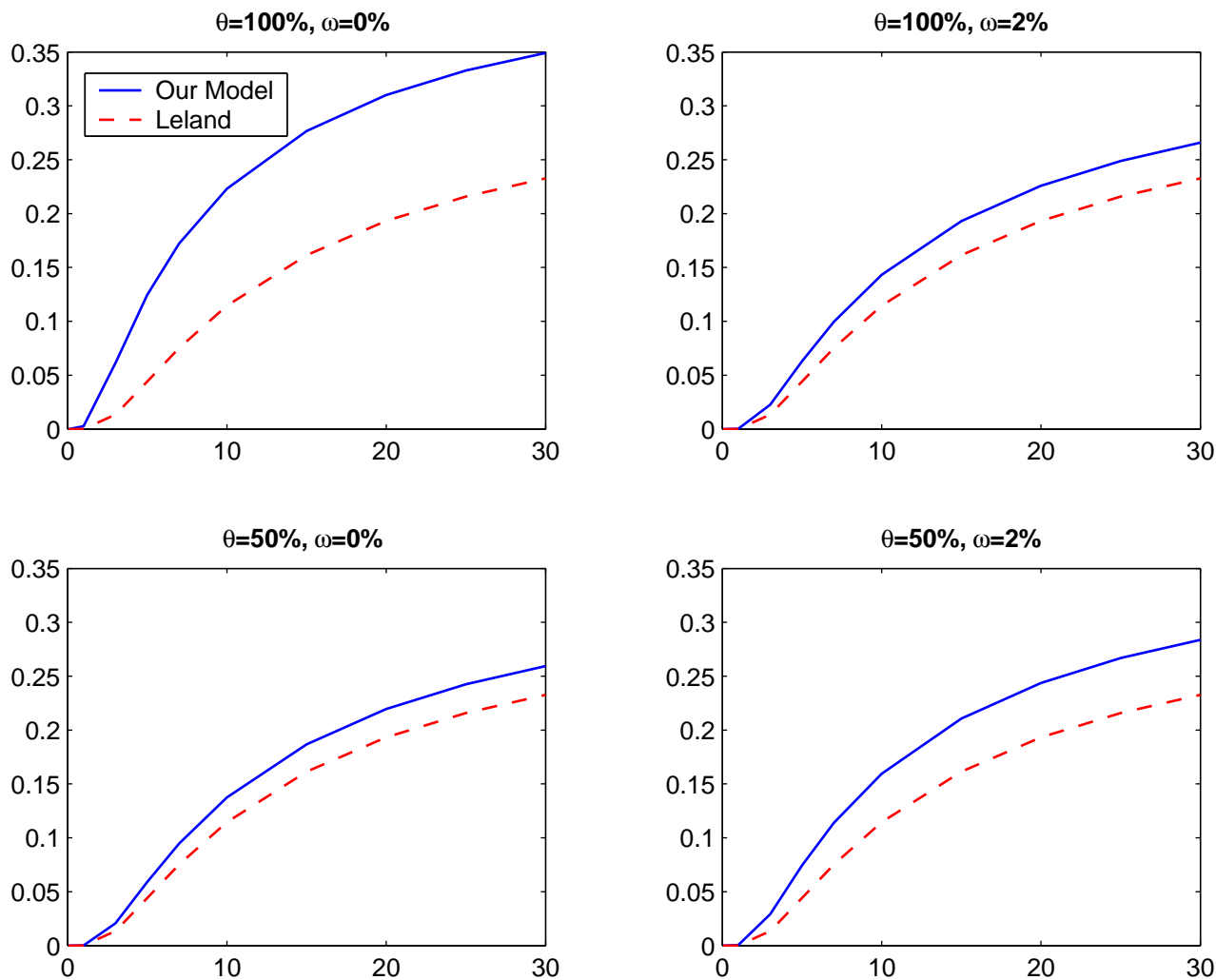
**Figure 4. Equity Value Maximization.** These plots correspond to the case when the bankruptcy boundary is selected to maximize the equity value. The panels show the optimal bankruptcy boundary, firm and equity values, and the credit spread as a percentage of the benchmark Leland (1994) model for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ . The values of other parameters are provided in Table I. The straight line in the upper left plot designates the illiquidity boundary ( $\delta_t = c$ ).



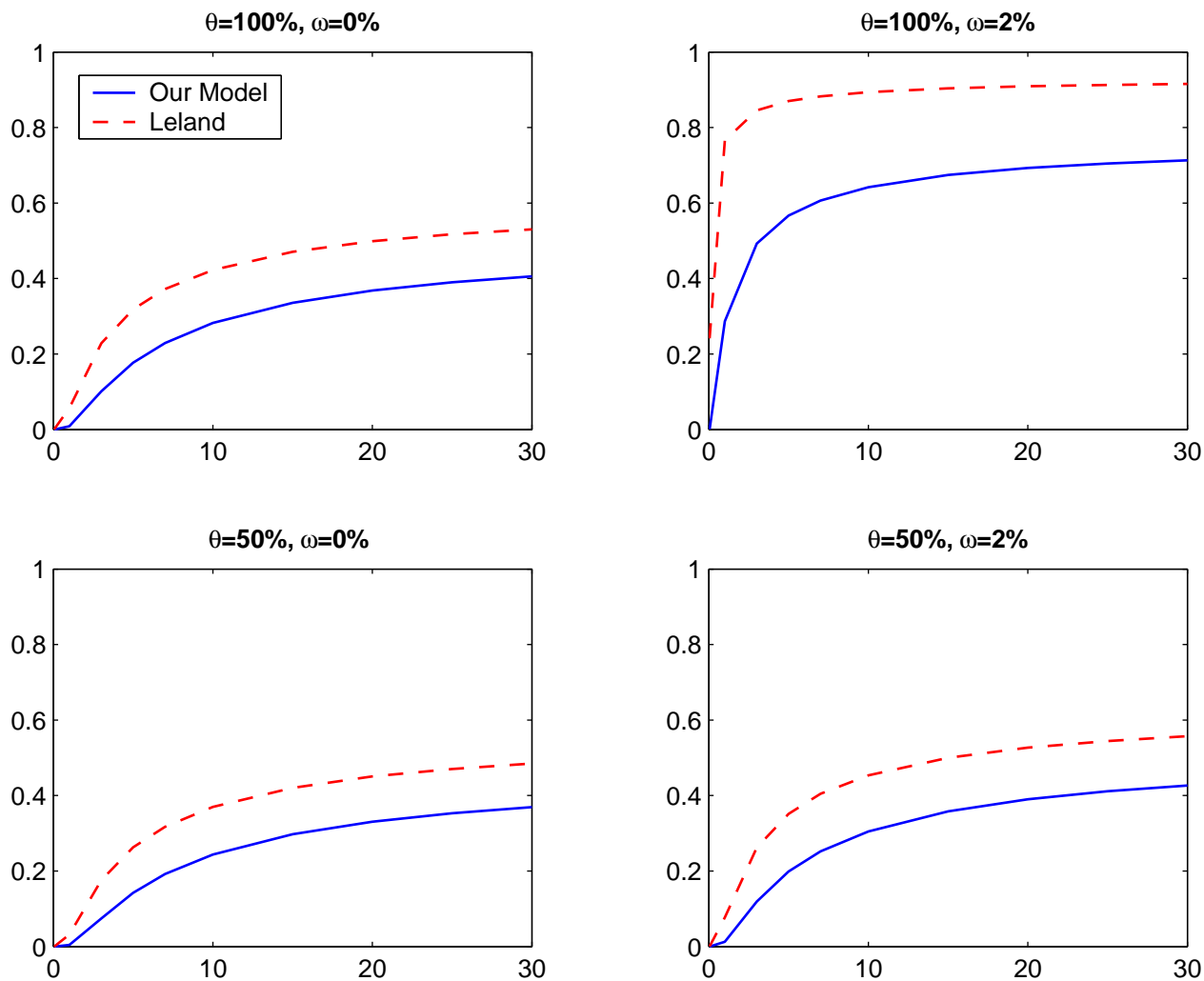
**Figure 5. Debt Value Maximization.** These plots correspond to the case when the bankruptcy boundary is selected to maximize the debt value. The panels show the optimal bankruptcy boundary, firm and equity values, and the credit spread as a percentage of the benchmark Leland (1994) model for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ . The values of other parameters are provided in Table I. The straight line in the upper left plot designates the illiquidity boundary ( $\delta_t = c$ ).



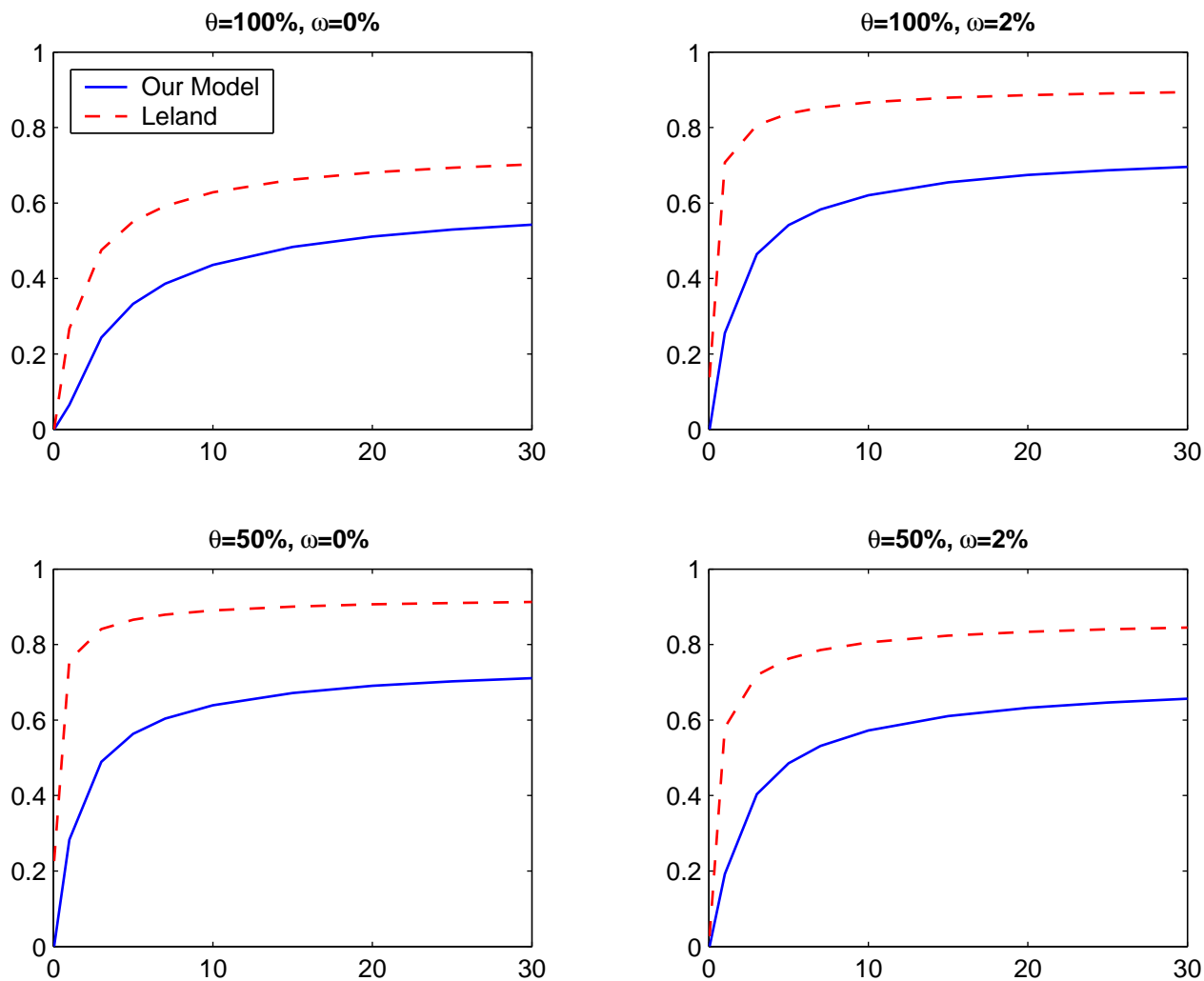
**Figure 6. Term Structure of Default Probabilities (Equity Maximization).** This figure shows the term structure of default probabilities,  $\mathbb{P}_B(t, c)$ , in our and Leland's (1994) models in the case when the bankruptcy boundary is selected to maximize the firm's equity value for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ , parameters. The values of other parameters are provided in Table I. The horizontal axis shows the default horizon,  $t$ , in years, the vertical axis show the probability of default over the corresponding horizon.



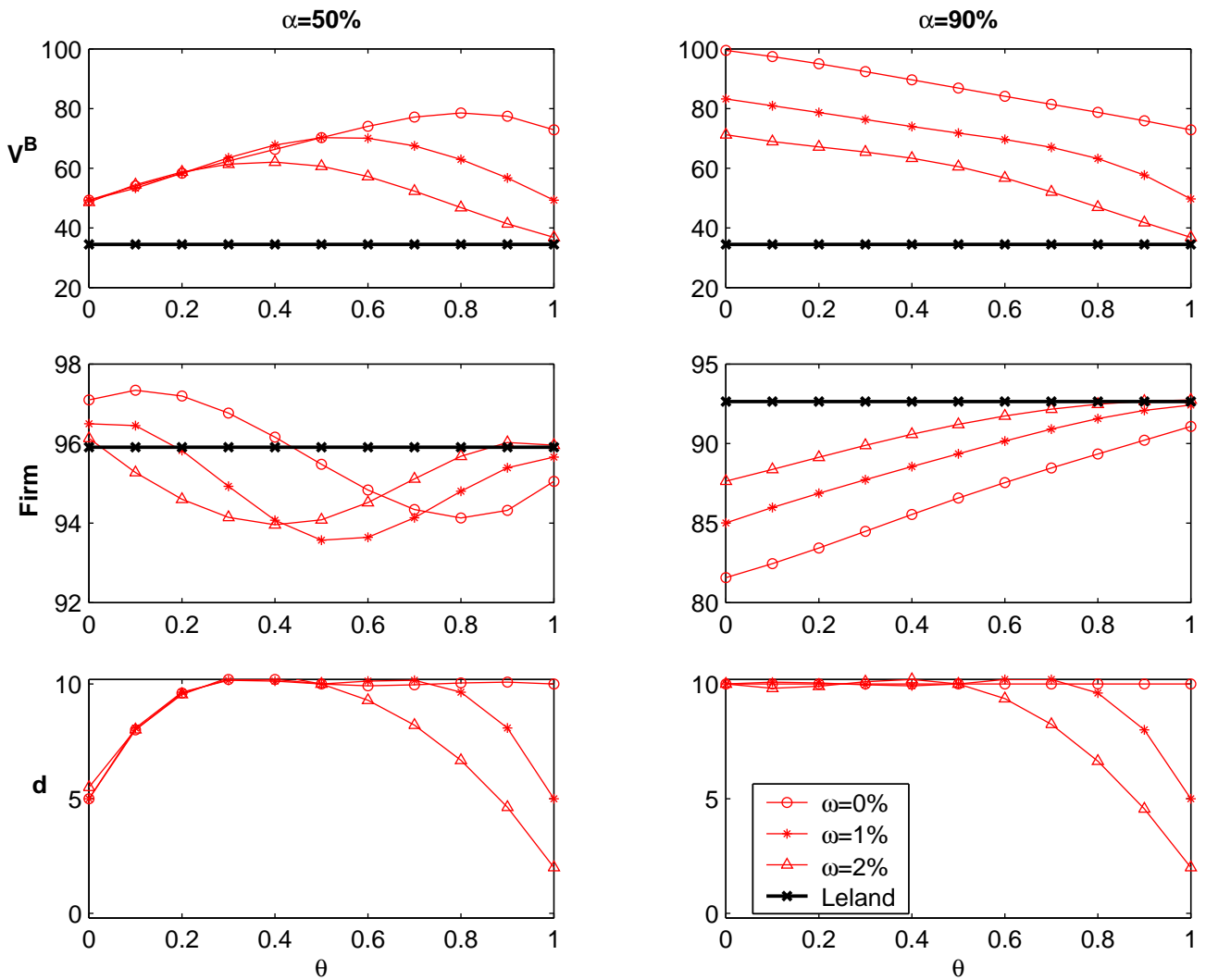
**Figure 7. Term Structure of Default Probabilities (Debt Maximization).** This figure shows the term structure of default probabilities,  $\mathbb{P}_B(t, c)$ , in our and Leland's (1994) models in the case when the bankruptcy boundary is selected to maximize the firm's debt value for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ , parameters. The values of other parameters are provided in Table I. The horizontal axis shows the default horizon,  $t$ , in years, the vertical axis show the probability of default over the corresponding horizon.



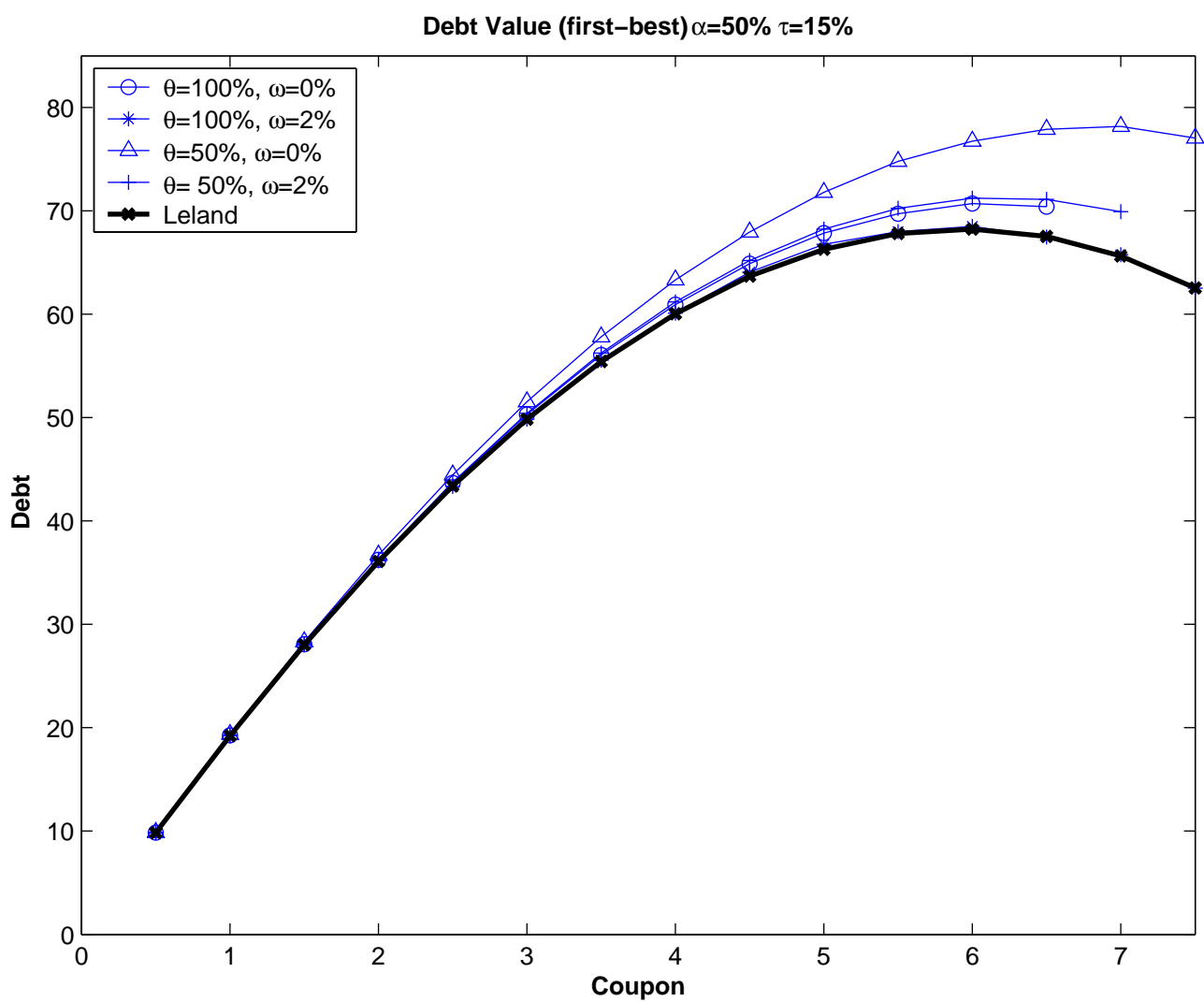
**Figure 8. Term Structure of Liquidation Probabilities (Equity Maximization).** This figure shows the term structure of liquidation probabilities,  $\mathbb{P}_L(t, \delta^B)$ , in our and Leland's (1994) models in the case when the bankruptcy boundary is selected to maximize the firm's equity value for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ , parameters. The values of other parameters are provided in Table I. The horizontal axis shows the default horizon,  $t$ , in years, the vertical axis show the probability of default over the corresponding horizon.



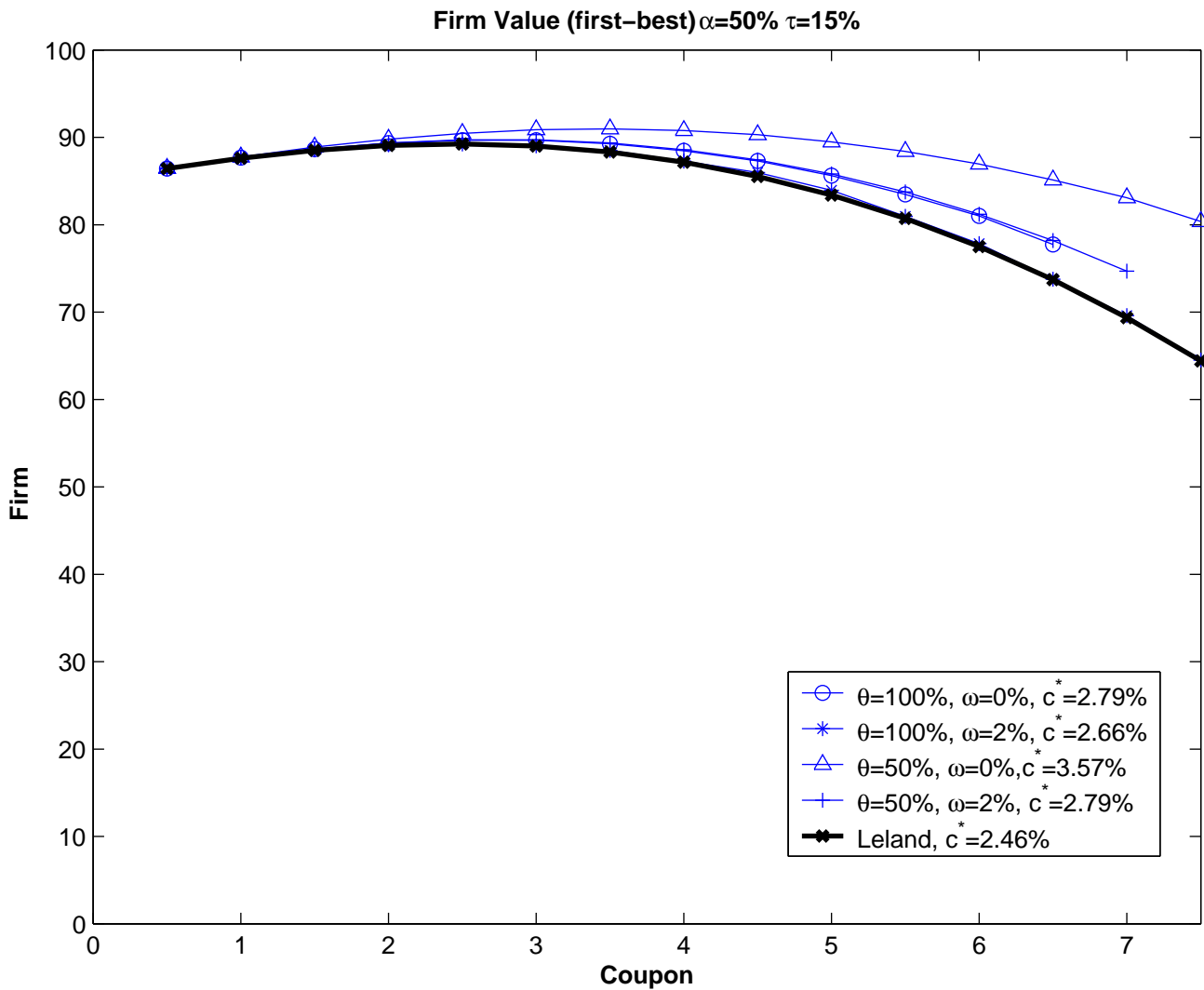
**Figure 9. Term Structure of Liquidation Probabilities (Debt Maximization).** This figure shows the term structure of liquidation probabilities,  $\mathbb{P}_L(t, \delta^B)$ , in our and Leland's (1994) models in the case when the bankruptcy boundary is selected to maximize the firm's debt value for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ , parameters. The values of other parameters are provided in Table I. The horizontal axis shows the default horizon,  $t$ , in years, the vertical axis show the probability of default over the corresponding horizon.



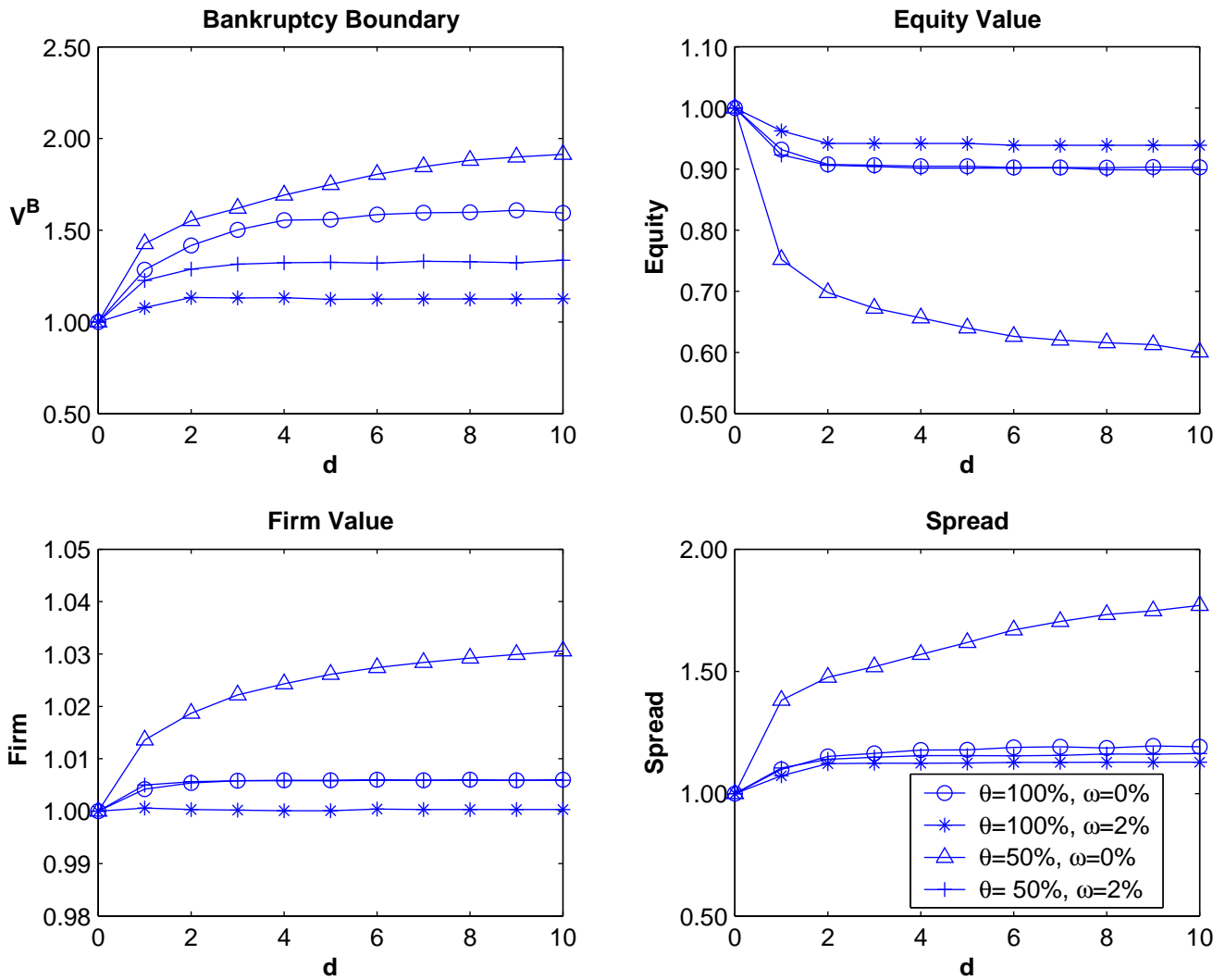
**Figure 10. Transfer of Control Rights.** We investigate the impact of the transfer of control from equity to debt in bankruptcy. We show the optimal bankruptcy boundary, overall firm value, and the respective optimal grace period  $d$  as a function of debt forgiveness  $\theta$  for various combinations of the liquidation cost  $\alpha$  and distress cost  $\omega$ . The values of other parameters are provided in Table I.



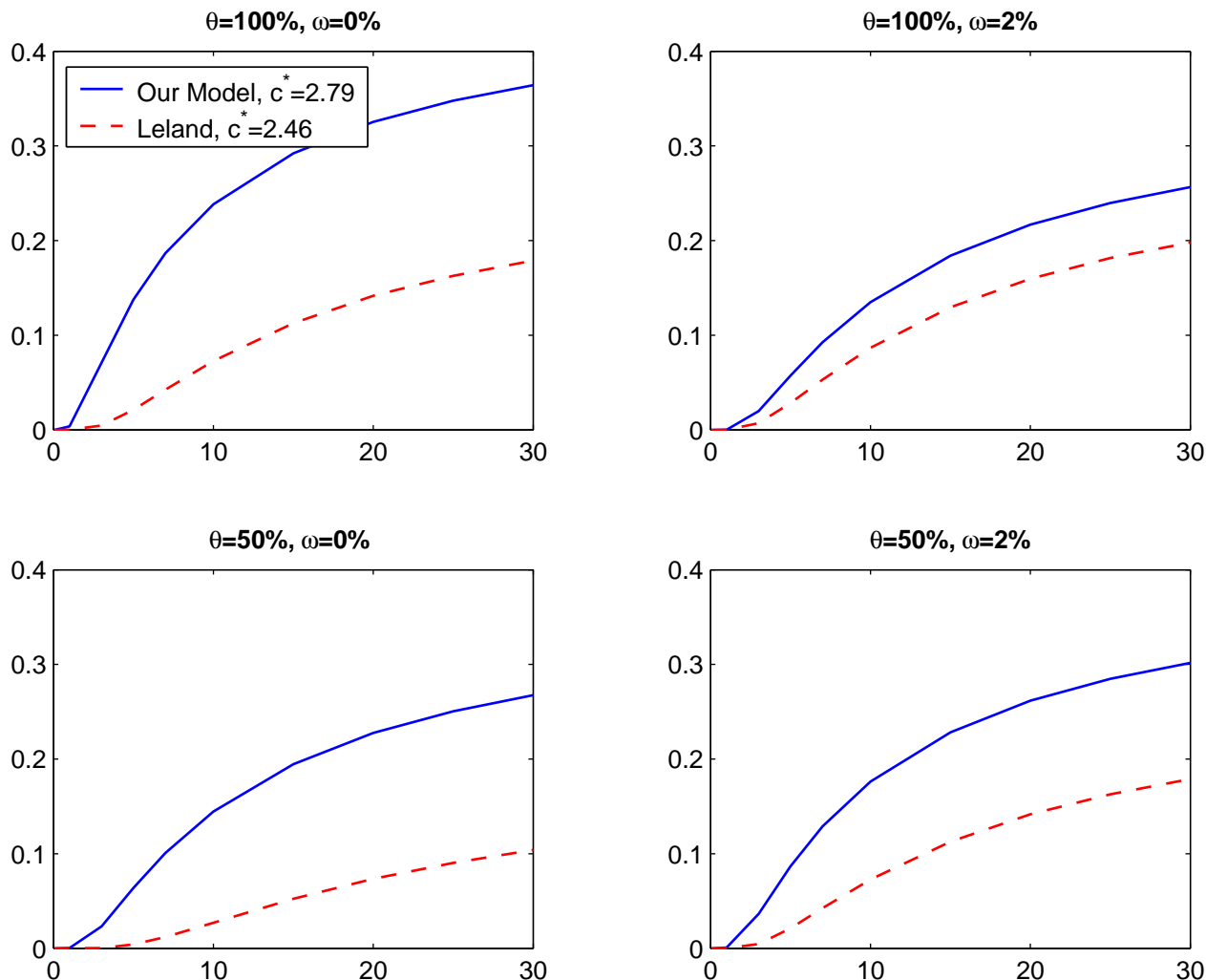
**Figure 11. Debt Capacity.** The figure shows the firm’s debt value as a function of the coupon rate for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ , parameters for the case when the bankruptcy boundary is selected to maximize the firm value. We assumed liquidation cost of  $\alpha = 50\%$ , grace period  $d = 2$  years and an effective tax rate of  $\tau = 15\%$ . The values of other parameters are provided in Table I. We see that debt capacity corresponding to the coupon rate is higher if the bankruptcy option is available.



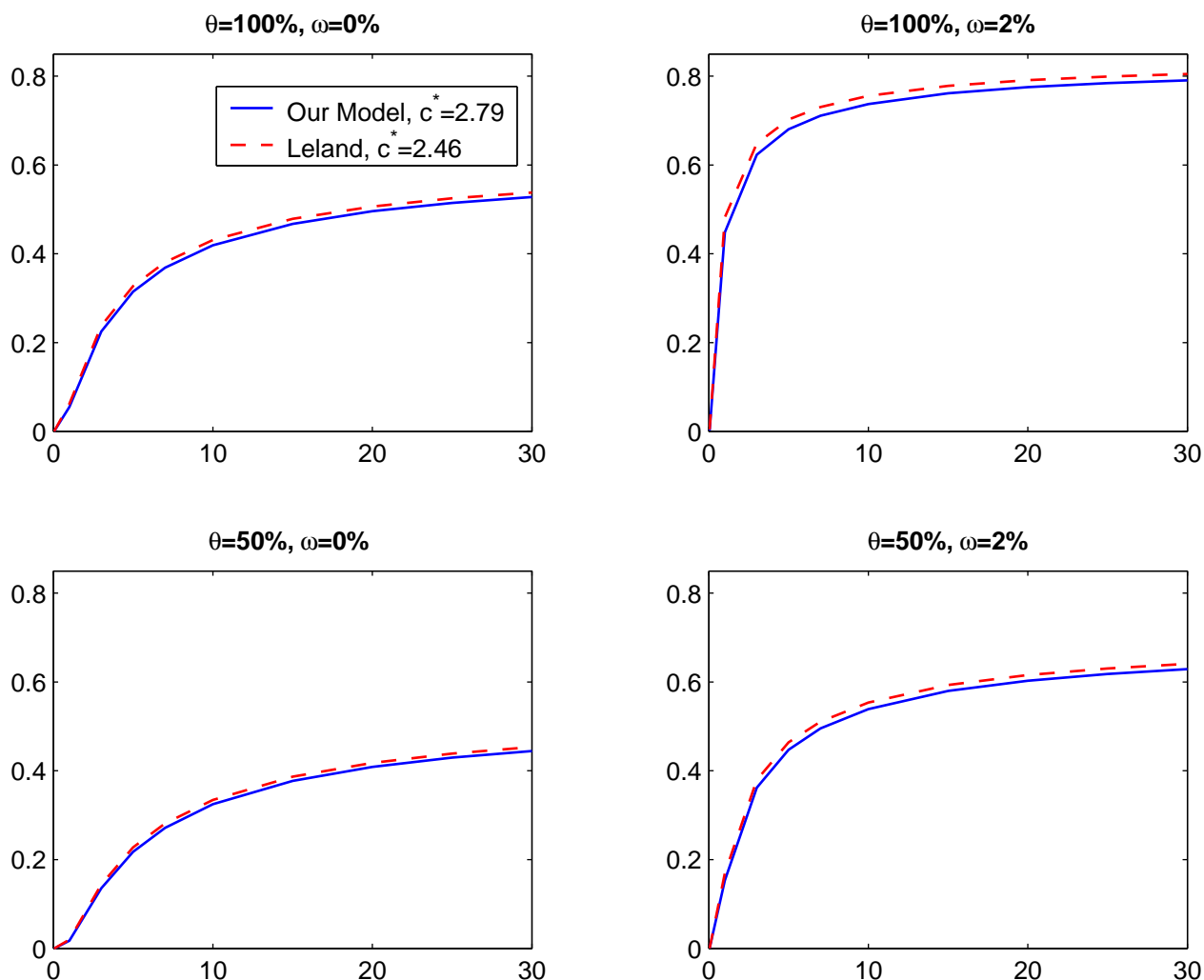
**Figure 12. Optimal Capital Structure.** In this figure we show the overall firm value as a function of the coupon rate for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ , parameters for the case when the bankruptcy boundary is selected to maximize the firm value. We assumed liquidation cost of  $\alpha = 50\%$ , grace period  $d = 2$  years and an effective tax rate of  $\tau = 15\%$ . The values of other parameters are provided in Table I. We see that the optimal capital structure involves higher level of debt (corresponding to the optimal coupon rate  $c^*$ ) if the bankruptcy option is available.



**Figure 13. Firm Value Maximization with Optimal Capital Structure.** These plots correspond to the case when the bankruptcy boundary is selected to maximize the overall firm value (first best) given that the coupon is selected to maximize the firm value as well. The panels show the optimal bankruptcy boundary, firm and equity values, and the credit spread as a percentage of the benchmark Leland (1994) model for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ . The values of other parameters are provided in Table I.



**Figure 14. Probability of default for optimal capital structure.** This figure shows the term structure of default probabilities,  $\mathbb{P}_B(t, c)$ , in our and Leland's (1994) models in the case when the bankruptcy boundary is selected to maximize the firm value for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ , parameters. The grace period is equal to  $d = 2$  years. The optimal coupon values,  $c^*$ , are reported in the plot's legend. The values of other parameters are provided in Table I. The horizontal axis shows the default horizon,  $t$ , in years, the vertical axis show the probability of default over the corresponding horizon.



**Figure 15. Probability of liquidation for optimal capital structure.** This figure shows the term structure of liquidation probabilities,  $\mathbb{P}_L(t, \delta^B)$ , in our and Leland's (1994) models in the case when the bankruptcy boundary is selected to maximize the firm value for various combinations of the debt forgiveness,  $\theta$ , and distress cost,  $\omega$ , parameters. The grace period is equal to  $d = 2$  years. The optimal coupon values,  $c^*$ , are reported in the plot's legend. The values of other parameters are provided in Table I. The horizontal axis shows the default horizon,  $t$ , in years, the vertical axis show the probability of default over the corresponding horizon.